

Consider the linear system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= -4x + 2y, \\ \frac{dy}{dt} &= x - 3y.\end{aligned}$$

1. Find the equilibrium solutions.
2. Write the system in matrix form  $\frac{dY}{dt} = AY$ .
3. Calculate the eigenvalues and eigenvectors of the matrix  $A$ .
4. Use the eigenvalues and eigenvectors to write down two linearly independent straight-line solutions to the system of differential equations.
5. Use the two linearly independent straight-line solutions to write the general solution to the system.
6. Find the particular solution satisfying the initial condition  $x(0) = 2$ ,  $y(0) = -4$ .
7. Using pplane or HPGSystemSolver or other suitable software, sketch the phase portrait of the system. On the phase portrait, include the two straight-line solutions from #4 and the particular solution from #6. Highlight and label these solutions. Also, mark the equilibrium solutions.
8. What happens to solutions as  $t$  gets large? Be as specific as you can.

