

Section 13.4. Consider a vibrating elastic string whose left end is free and whose right end is fixed. (“Free” means that the end is attached to a frictionless vertical track and is free to move up and down.) The corresponding initial-boundary-value problem is

$$\begin{aligned}\frac{\partial^2 u}{\partial t^2} &= c^2 \frac{\partial^2 u}{\partial x^2}, & 0 < x < L, & \quad t > 0, \\ \frac{\partial u}{\partial x}(0, t) &= 0 = u(L, t), & t > 0, \\ u(x, 0) &= f(x), & \frac{\partial u}{\partial t}(x, 0) &= g(x), & 0 < x < L.\end{aligned}$$

Solve this initial-boundary-value problem for

$$L = \frac{\pi}{2}, \quad f(x) = 1 - \frac{2x}{\pi}, \quad g(x) = 0.$$