

Robotics: From Basic Terminology to Monte Carlo Localization

Todd W. Neller

Figures © S. Russell, P. Norvig. *Artificial Intelligence: A Modern Approach*
or S. Thrun, W. Burgard, D. Fox. *Probabilistic Robotics* except where otherwise noted.

Robotics

- Robot – physical agent that performs tasks by sensing and manipulating the physical world
 - Sensors – input sensor data from environment for state estimation
 - Effectors – output control actions for actuating effectors to change state and/or environment
 - Maximizing expected utility – generate and execute control outputs that affect change so as to maximize rewards
- Environment complexity: partially observable, stochastic, people acting in environment (prediction needed), continuous state and action spaces, sometimes high-dimensional, real-time
- Outside of computation, most aspects of robotics struggle with the challenges of ***uncertainty***.

Localization

- “Where am I?” “What’s the situation?”
- Given:
 - Initial pose (state)
 - Environment map
 - Sensor/measurement model
 - Motion model
- Keep track of the most likely current state as the robot moves and senses in the environment.

Monte Carlo Localization

- Monte Carlo Localization
 - Initial set of random hypotheses about possible poses (i.e. states, particles)
 - As the robot moves and senses, a Darwinian survival-of-the-fittest process tends to reproduce the most likely hypotheses and tends to kill off the least likely.
 - Evolution of a cloud of hypotheses where the center, i.e. average, is the most likely robot pose.

MCL Algorithm



```
1: Algorithm MCL( $\mathcal{X}_{t-1}, u_t, z_t, m$ ):  
2:    $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$   
3:   for  $m = 1$  to  $M$  do  
4:      $x_t^{[m]} = \text{sample\_motion\_model}(u_t, x_{t-1}^{[m]})$   
5:      $w_t^{[m]} = \text{measurement\_model}(z_t, x_t^{[m]}, m)$   
6:      $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$   
7:   endfor  
8:   for  $m = 1$  to  $M$  do  
9:     draw  $i$  with probability  $\propto w_t^{[i]}$   
10:    add  $x_t^{[i]}$  to  $\mathcal{X}_t$   
11:  endfor  
12:  return  $\mathcal{X}_t$ 
```



NOTE: These two m variables refer to the environment map, and not to the for-loop control variable that serves as an index to the poses/weights.

Figure 1: Pseudocode for MCL (Thrun, Burgard, and Fox 2005)

Motion Model

- u_t – motion control command at time $t - 1$ to affect time t
- x_{t-1} – pose at time $t - 1$
- x_t – pose at time t
- Model $p(x_t | u_t, x_{t-1})$ – the probability of the pose being x_t given prior motion control command u_t and prior pose x_{t-1}

$$p(x_t | u_t, x_{t-1})$$

Example: 2D Robot with Rotation

- Simple resting pose (state): $(x, y, \theta)^T$
- Control translational and rotational velocities: $(v, \omega)^T$
- Error parameters: $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$

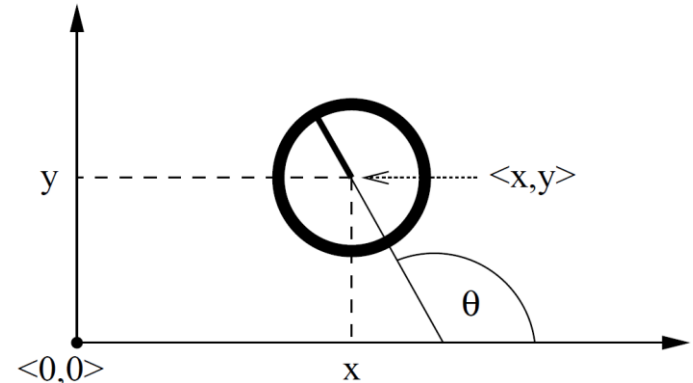


Figure 5.1 Robot pose, shown in a global coordinate system.

Motion Model Based on Velocity Command

$$x_{t-1} = (x, y, \theta)^T$$

$$x_t = (x', y', \theta')^T$$

1:	Algorithm <code>motion_model_velocity</code> (x_t, u_t, x_{t-1}):
2:	$\mu = \frac{1}{2} \frac{(x - x') \cos \theta + (y - y') \sin \theta}{(y - y') \cos \theta - (x - x') \sin \theta}$
3:	$x^* = \frac{x + x'}{2} + \mu(y - y')$
4:	$y^* = \frac{y + y'}{2} + \mu(x' - x)$
5:	$r^* = \sqrt{(x - x^*)^2 + (y - y^*)^2}$
6:	$\Delta\theta = \text{atan2}(y' - y^*, x' - x^*) - \text{atan2}(y - y^*, x - x^*)$
7:	$\hat{v} = \frac{\Delta\theta}{\Delta t} r^*$
8:	$\hat{\omega} = \frac{\Delta\theta}{\Delta t}$
9:	$\hat{\gamma} = \frac{\theta' - \theta}{\Delta t} - \hat{\omega}$
10:	$\text{return } \mathbf{prob}(v - \hat{v}, \alpha_1 v^2 + \alpha_2 \omega^2) \cdot \mathbf{prob}(\omega - \hat{\omega}, \alpha_3 v^2 + \alpha_4 \omega^2) \\ \cdot \mathbf{prob}(\hat{\gamma}, \alpha_5 v^2 + \alpha_6 \omega^2)$

Table 5.1 Algorithm for computing $p(x_t | u_t, x_{t-1})$ based on velocity information. Here we assume x_{t-1} is represented by the vector $(x \ y \ \theta)^T$; x_t is represented by $(x' \ y' \ \theta')^T$; and u_t is represented by the velocity vector $(v \ \omega)^T$. The function $\mathbf{prob}(a, b^2)$ computes the probability of its argument a under a zero-centered distribution with variance b^2 . It may be implemented using any of the algorithms in Table 5.2.

Motion Model Based on Velocity Command

$$x_{t-1} = (x, y, \theta)^T$$

$$x_t = (x', y', \theta')^T$$

Turning radius
circle position
and radius

```

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```

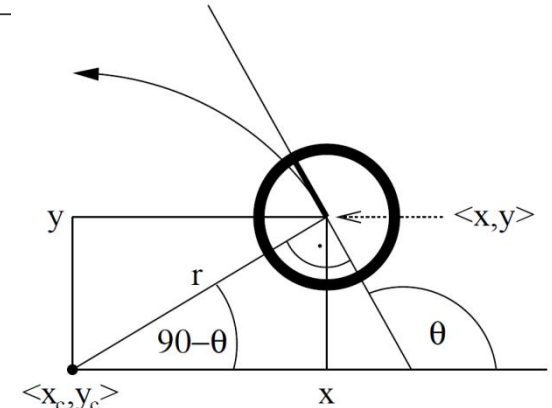


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Turning radius
circle position
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Change of
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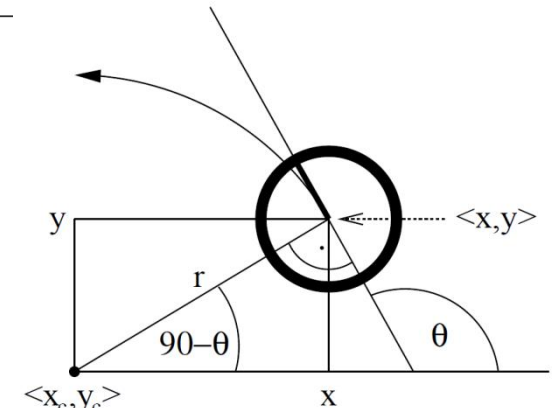


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circle position
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Change of
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Translational
and rotational
velocity,
final rotation
(error)

1: **Algorithm motion_model_velocity(x_t, u_t, x_{t-1}):**

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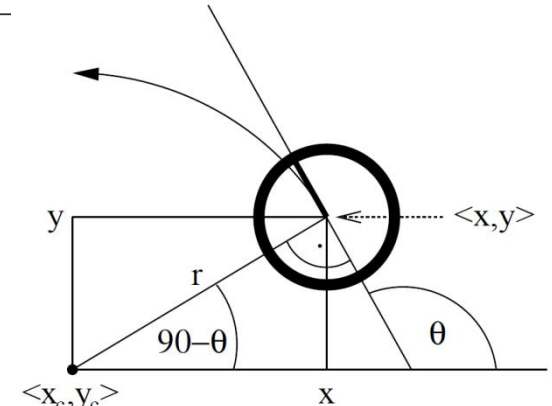


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Probabilities of discrepancies between commanded and computed velocities & rotation, given error parameters times velocity magnitudes

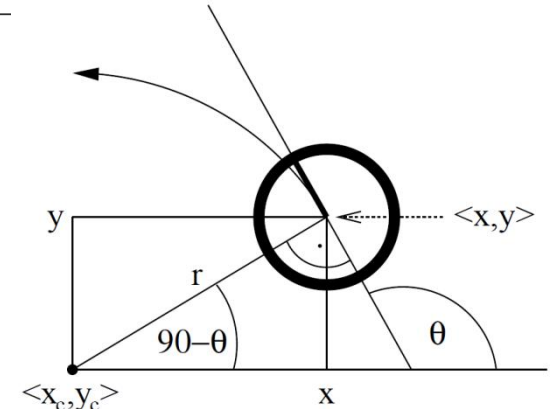


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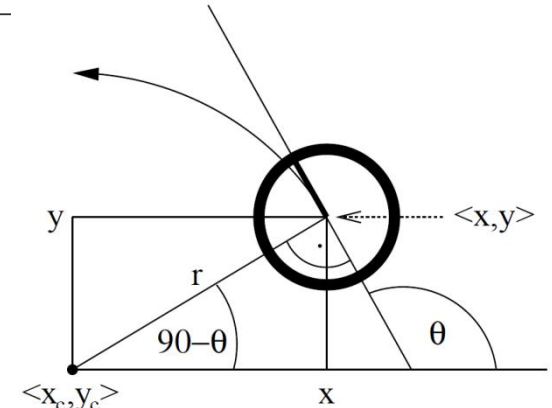
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Probabilities of discrepancies between commanded and computed velocities & rotation, given error parameters times velocity magnitudes



But what is
prob ?

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Common Probability Distributions

prob is a zero-mean probability distribution with variance b . Examples:

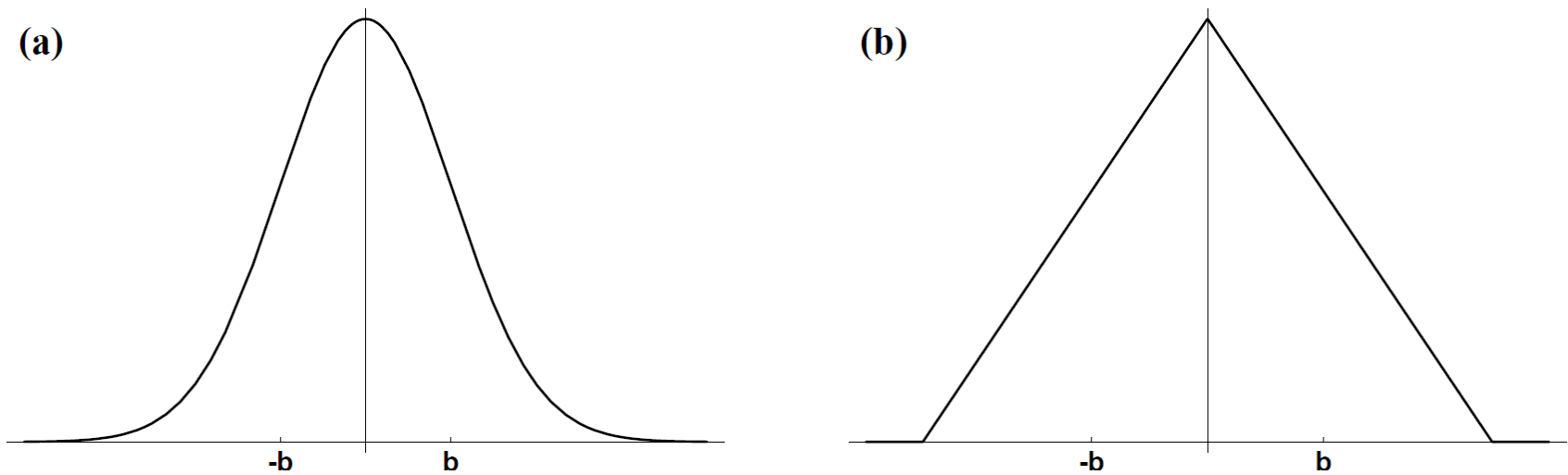


Figure 5.6 Probability density functions with variance b : (a) Normal distribution, (b) triangular distribution.

Computing Prob

1: **Algorithm prob_normal_distribution(a, b^2):**

2: *return* $\frac{1}{\sqrt{2\pi} b^2} \exp \left\{ -\frac{1}{2} \frac{a^2}{b^2} \right\}$

3: **Algorithm prob_triangular_distribution(a, b^2):**

4: *return* $\max \left\{ 0, \frac{1}{\sqrt{6} b} - \frac{|a|}{6 b^2} \right\}$

Table 5.2 Algorithms for computing densities of a zero-centered normal distribution and a triangular distribution with variance b^2 .

$\text{prob}_X(a, b^2)$ is the probability of a occurring for the zero-centered distribution X with variance b^2

Simulating Particles with Error Sampling

```
1:   Algorithm sample_motion_model_velocity( $u_t, x_{t-1}$ ):  
2:        $\hat{v} = v + \text{sample}(\alpha_1 v^2 + \alpha_2 \omega^2)$   
3:        $\hat{\omega} = \omega + \text{sample}(\alpha_3 v^2 + \alpha_4 \omega^2)$   
4:        $\hat{\gamma} = \text{sample}(\alpha_5 v^2 + \alpha_6 \omega^2)$   
5:        $x' = x - \frac{\hat{v}}{\hat{\omega}} \sin \theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega} \Delta t)$   
6:        $y' = y + \frac{\hat{v}}{\hat{\omega}} \cos \theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega} \Delta t)$   
7:        $\theta' = \theta + \hat{\omega} \Delta t + \hat{\gamma} \Delta t$   
8:       return  $x_t = (x', y', \theta')^T$ 
```

Table 5.3 Algorithm for sampling poses $x_t = (x' \ y' \ \theta')^T$ from a pose $x_{t-1} = (x \ y \ \theta)^T$ and a control $u_t = (v \ \omega)^T$. Note that we are perturbing the final orientation by an additional random term, $\hat{\gamma}$. The variables α_1 through α_6 are the parameters of the motion noise. The function `sample(b^2)` generates a random sample from a zero-centered distribution with variance b^2 . It may, for example, be implemented using the algorithms in Table 5.4.

Sampling with Normal and Triangular Distributions

1: **Algorithm `sample_normal_distribution(b2)`:**

2:
$$\text{return } \frac{1}{2} \sum_{i=1}^{12} \text{rand}(-b, b)$$

`rand(-b,b) in Java: b * (Math.random() + Math.random() - 1)`

3: **Algorithm `sample_triangular_distribution(b2)`:**

4:
$$\text{return } \frac{\sqrt{6}}{2} [\text{rand}(-b, b) + \text{rand}(-b, b)]$$

`Java: return (Math.random() + Math.random() - 1) * SCALE;`
`where final double SCALE = Math.sqrt(6 * variance);`

Table 5.4 Algorithm for sampling from (approximate) normal and triangular distributions with zero mean and variance b^2 ; see Winkler (1995: p293). The function **rand**(x, y) is assumed to be a pseudo random number generator with uniform distribution in $[x, y]$.

In General

- Collect a lot of motion data and use that data to create your motion model.

Measurement Model

- z_t – sensor inputs at time t
- x_t – pose (state) at time t
- m – map of the environment
- $p(z_t | x_t, m)$ – probability of measuring z_t given pose x_t and map m

$$p(z_t | x_t, m)$$

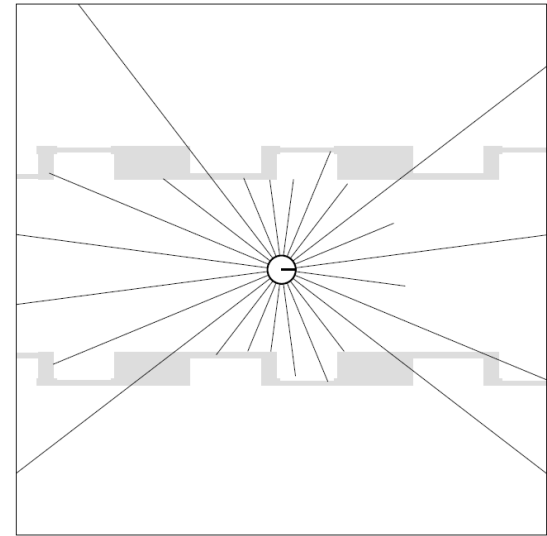


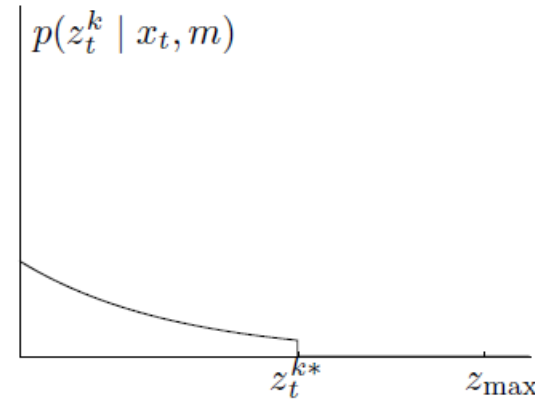
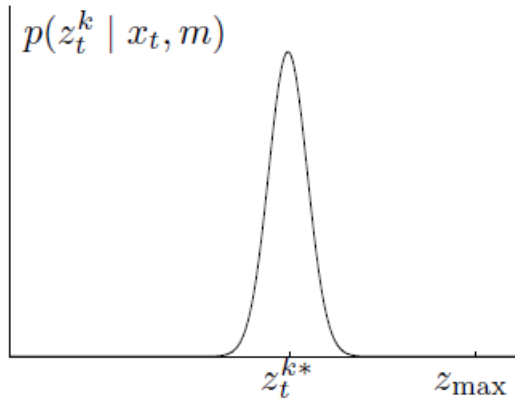
Figure 6.1 Typical ultrasound scan of a robot in its environment.

Distributions for Modeling Different Kinds of Error Expectations

(a) Gaussian distribution p_{hit}

(b) Exponential distribution p_{short}

Expected variation around true object range value

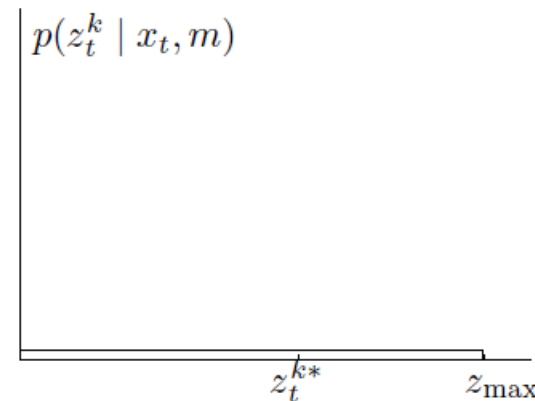
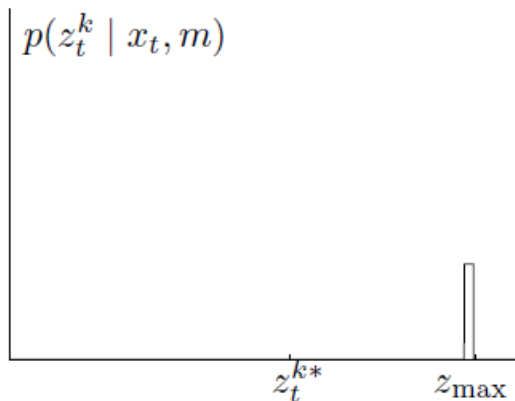


Unexpected closer objects

(c) Uniform distribution p_{max}

(d) Uniform distribution p_{rand}

Object missed by range finder and max range returned



General unexplained sensor noise

Figure 6.3 Components of the range finder sensor model. In each diagram the horizontal axis corresponds to the measurement z_t^k , the vertical to the likelihood.

Added together...

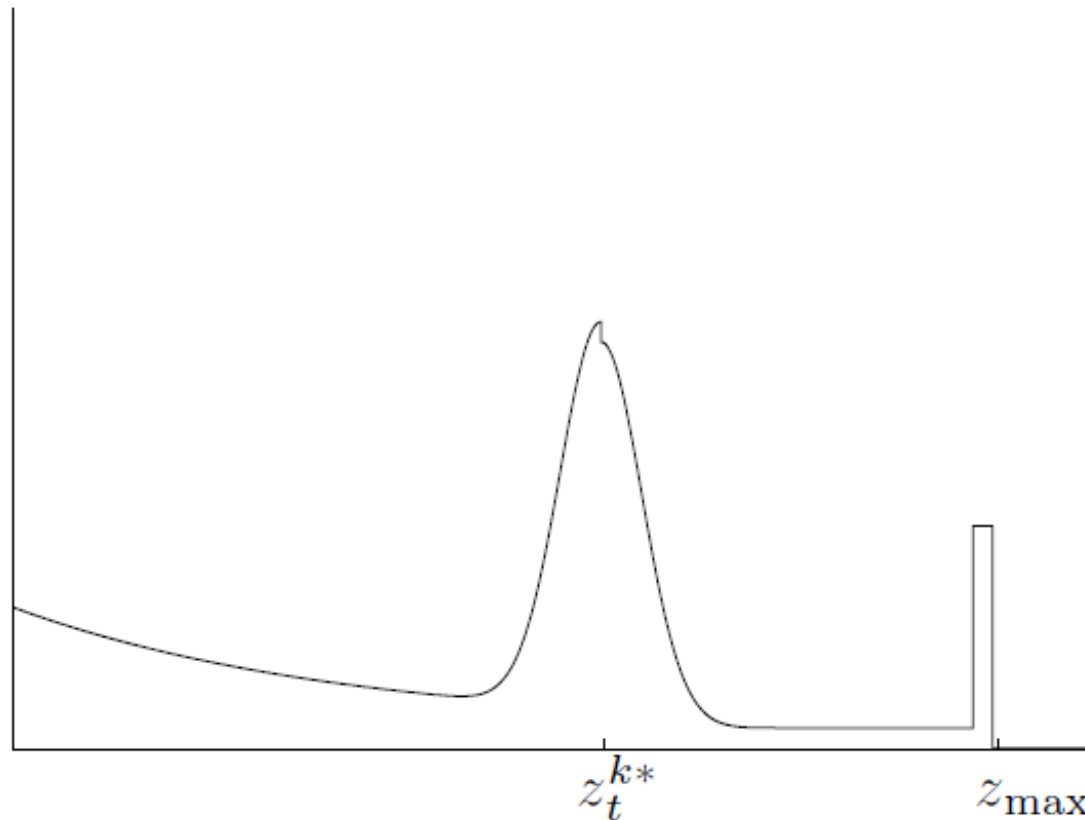


Figure 6.4 “Pseudo-density” of a typical mixture distribution $p(z_t^k | x_t, m)$.

... To Make a Weighted Average ...

These four different distributions are now mixed by a weighted average, defined by the parameters z_{hit} , z_{short} , z_{max} , and z_{rand} with $z_{\text{hit}} + z_{\text{short}} + z_{\text{max}} + z_{\text{rand}} = 1$.

$$(6.12) \quad p(z_t^k | x_t, m) = \begin{pmatrix} z_{\text{hit}} \\ z_{\text{short}} \\ z_{\text{max}} \\ z_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z_t^k | x_t, m) \\ p_{\text{short}}(z_t^k | x_t, m) \\ p_{\text{max}}(z_t^k | x_t, m) \\ p_{\text{rand}}(z_t^k | x_t, m) \end{pmatrix}$$

```
1:   Algorithm beam_range_finder_model( $z_t, x_t, m$ ):
2:        $q = 1$ 
3:       for  $k = 1$  to  $K$  do
4:           compute  $z_t^{k*}$  for the measurement  $z_t^k$  using ray casting
5:            $p = z_{\text{hit}} \cdot p_{\text{hit}}(z_t^k | x_t, m) + z_{\text{short}} \cdot p_{\text{short}}(z_t^k | x_t, m)$ 
6:                $+ z_{\text{max}} \cdot p_{\text{max}}(z_t^k | x_t, m) + z_{\text{rand}} \cdot p_{\text{rand}}(z_t^k | x_t, m)$ 
7:            $q = q \cdot p$ 
8:       return  $q$ 
```

Table 6.1 Algorithm for computing the likelihood of a range scan z_t , assuming conditional independence between the individual range measurements in the scan.

... But How Should We Set the Weights?

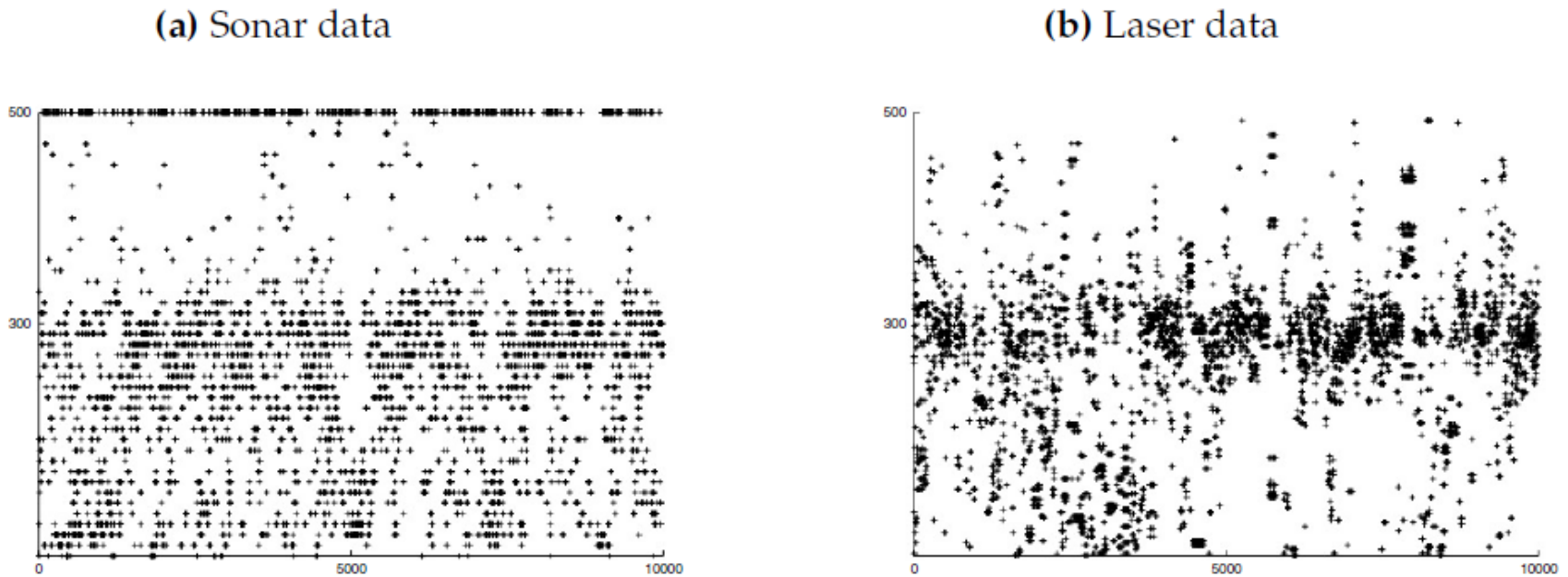


Figure 6.5 Typical data obtained with (a) a sonar sensor and (b) a laser-range sensor in an office environment for a “true” range of 300 cm and a maximum range of 500 cm.

Maximum Likelihood Estimation

- Iteratively tune parameters until the data distribution collected for known distance measurements achieves maximum likelihood.
- (Or manually tune for good match to data.)

```
1:  Algorithm learn_intrinsic_parameters( $Z, X, m$ ):
2:      repeat until convergence criterion satisfied
3:          for all  $z_i$  in  $Z$  do
4:               $\eta = [ p_{\text{hit}}(z_i | x_i, m) + p_{\text{short}}(z_i | x_i, m)$ 
                     $+ p_{\text{max}}(z_i | x_i, m) + p_{\text{rand}}(z_i | x_i, m) ]^{-1}$ 
5:              calculate  $z_i^*$ 
6:               $e_{i,\text{hit}} = \eta p_{\text{hit}}(z_i | x_i, m)$ 
7:               $e_{i,\text{short}} = \eta p_{\text{short}}(z_i | x_i, m)$ 
8:               $e_{i,\text{max}} = \eta p_{\text{max}}(z_i | x_i, m)$ 
9:               $e_{i,\text{rand}} = \eta p_{\text{rand}}(z_i | x_i, m)$ 
10:              $z_{\text{hit}} = |Z|^{-1} \sum_i e_{i,\text{hit}}$ 
11:              $z_{\text{short}} = |Z|^{-1} \sum_i e_{i,\text{short}}$ 
12:              $z_{\text{max}} = |Z|^{-1} \sum_i e_{i,\text{max}}$ 
13:              $z_{\text{rand}} = |Z|^{-1} \sum_i e_{i,\text{rand}}$ 
14:              $\sigma_{\text{hit}} = \sqrt{\frac{1}{\sum_i e_{i,\text{hit}}} \sum_i e_{i,\text{hit}} (z_i - z_i^*)^2}$ 
15:              $\lambda_{\text{short}} = \frac{\sum_i e_{i,\text{short}}}{\sum_i e_{i,\text{short}} z_i}$ 
16:             return  $\Theta = \{z_{\text{hit}}, z_{\text{short}}, z_{\text{max}}, z_{\text{rand}}, \sigma_{\text{hit}}, \lambda_{\text{short}}\}$ 
```

Table 6.2 Algorithm for learning the intrinsic parameters of the beam-based sensor model from data.

1: **Algorithm learn_intrinsic_parameters(Z, X, m):**

2: *repeat until convergence criterion satisfied*

3: *for all z_i in Z do*

4:
$$\eta = [p_{\text{hit}}(z_i | x_i, m) + p_{\text{short}}(z_i | x_i, m) + p_{\text{max}}(z_i | x_i, m) + p_{\text{rand}}(z_i | x_i, m)]$$

5: *calculate z_i^**

6:
$$e_{i,\text{hit}} = \eta p_{\text{hit}}(z_i | x_i, m)$$

7:
$$e_{i,\text{short}} = \eta p_{\text{short}}(z_i | x_i, m)$$

8:
$$e_{i,\text{max}} = \eta p_{\text{max}}(z_i | x_i, m)$$

9:
$$e_{i,\text{rand}} = \eta p_{\text{rand}}(z_i | x_i, m)$$

10:
$$z_{\text{hit}} = |Z|^{-1} \sum_i e_{i,\text{hit}}$$

11:
$$z_{\text{short}} = |Z|^{-1} \sum_i e_{i,\text{short}}$$

12:
$$z_{\text{max}} = |Z|^{-1} \sum_i e_{i,\text{max}}$$

13:
$$z_{\text{rand}} = |Z|^{-1} \sum_i e_{i,\text{rand}}$$

14:
$$\sigma_{\text{hit}} = \sqrt{\frac{1}{\sum_i e_{i,\text{hit}}} \sum_i e_{i,\text{hit}} (z_i - z_i^*)^2}$$

15:
$$\lambda_{\text{short}} = \frac{\sum_i e_{i,\text{short}}}{\sum_i e_{i,\text{short}} z_i}$$

16: *return $\Theta = \{z_{\text{hit}}, z_{\text{short}}, z_{\text{max}}, z_{\text{rand}}, \sigma_{\text{hit}}, \lambda_{\text{short}}\}$*

Compute the relative likelihood of each measurement interpretation for each measurement.

Think of these as normalized weights for the interpretation of each measurement.

Set the new weights for the weighted sum according to their average relative likelihood

Also compute the most likely parameters for our distributions.

In General

- Collect a lot of measurement data and use that data to create your measurement model.

Project Goal: Monte Carlo Localization for the Kidnapped Robot Problem

- Goals:
 - Acquire a map of the environment, e.g. FASTSLAM or other appropriate techniques.
 - Implement Monte Carlo localization.
 - Solve the Kidnapped Robot Problem:
 - An autonomous robot is transported to an unknown state and must localize.
 - Optional:
 - Designate a robot home state.
 - Have the robot return home after being kidnapped and successfully localizing.

Project Tips

- Set simple goals. Follow the [KISS Principle](#). (You can always set more ambitious goals if you achieve these early.) Example:
 - 1D state space:
 - Have a fixed robot that can rotate range finder(s), a camera, or other localizing sensor to determine state θ .
 - When you start the system, let initial state θ_0 be the “home state”.
 - Have the system rotate and sense to build a mapping from sensor inputs to probable locations.
 - After it has terminated mapping, put it in a new mode seeking to return home when it does not appear to be home.
 - “Kidnap” it by rotating the robot and demonstrate that it can relocalize and return home.
- Divide labor: project lead, documentation, version control, sensor model, motor model, etc.
- Plan team meeting times in advance. Budget for 18 total hours for each over 2 weeks beyond class. Log hours.

Project Platform: Anki Cozmo

- Programming tools:
 - Cozmo SDK: <https://www.anki.com/en-us/cozmo/SDK>
 - cozmo-tools: <https://github.com/touretzkyds/cozmo-tools>
- Possible projects:
 1. Create a new project where the robot is restricted to rotational movement only and uses visual camera sensing to localize.
 2. Find and build upon prior Cozmo 2D localization and mapping work you might find.

