

# Games and Computation Homework #13: Regret Matching, Extensive Form, and Counterfactual Regret Minimization

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Answer these questions within the HW #13 Moodle quiz:

## Battle of the Games

P1\P2	B	V
B	1, 3	1, 0
V	1, 0	2, 1

Player 1 and player 2 like different games. Player 1 prefers to go out and play billiards. Player 2 prefers to play video games. Suppose they must choose an activity from {B (billiards), V (video game)} simultaneously without communication. Player 2 is indifferent between going out to play billiards and staying home to play video games alone, but prefers to play video games with player 1. If they disagree on their chosen activity, player 2 gets the utility of playing video games alone, and player 1 calls the evening a zero. Suppose that two players are performing simple regret matching as demonstrated in RegretMatchingDemo.java for the Battle of the Games.

### Regret Matching Strategy from Cumulative Regret 1

For player 1, the current regret sum, i.e. cumulative regret, is [-4.0 0.0]. What is the probability that player 1 will play B on the next iteration? \_\_\_\_\_

### Regret Matching Strategy from Cumulative Regret 2

For player 2, the current regret sum, i.e. cumulative regret, is [-4.0 1.0]. What is the probability that player 2 will play B on the next iteration? \_\_\_\_\_

### Regret Matching Strategy Action Regret 1

Suppose that on the next time step, players 1 and 2 choose B and V respectively. How much does player 1 regret not having played V? \_\_\_\_\_

### Regret Matching Strategy Action Regret 2

How much does player 2 regret not having played B? \_\_\_\_\_

### Strategy from Cumulative Regret 1 Part 2

After these regrets are accumulated to the regret sum, what is the probability that player 1 will play B on the next iteration? \_\_\_\_\_

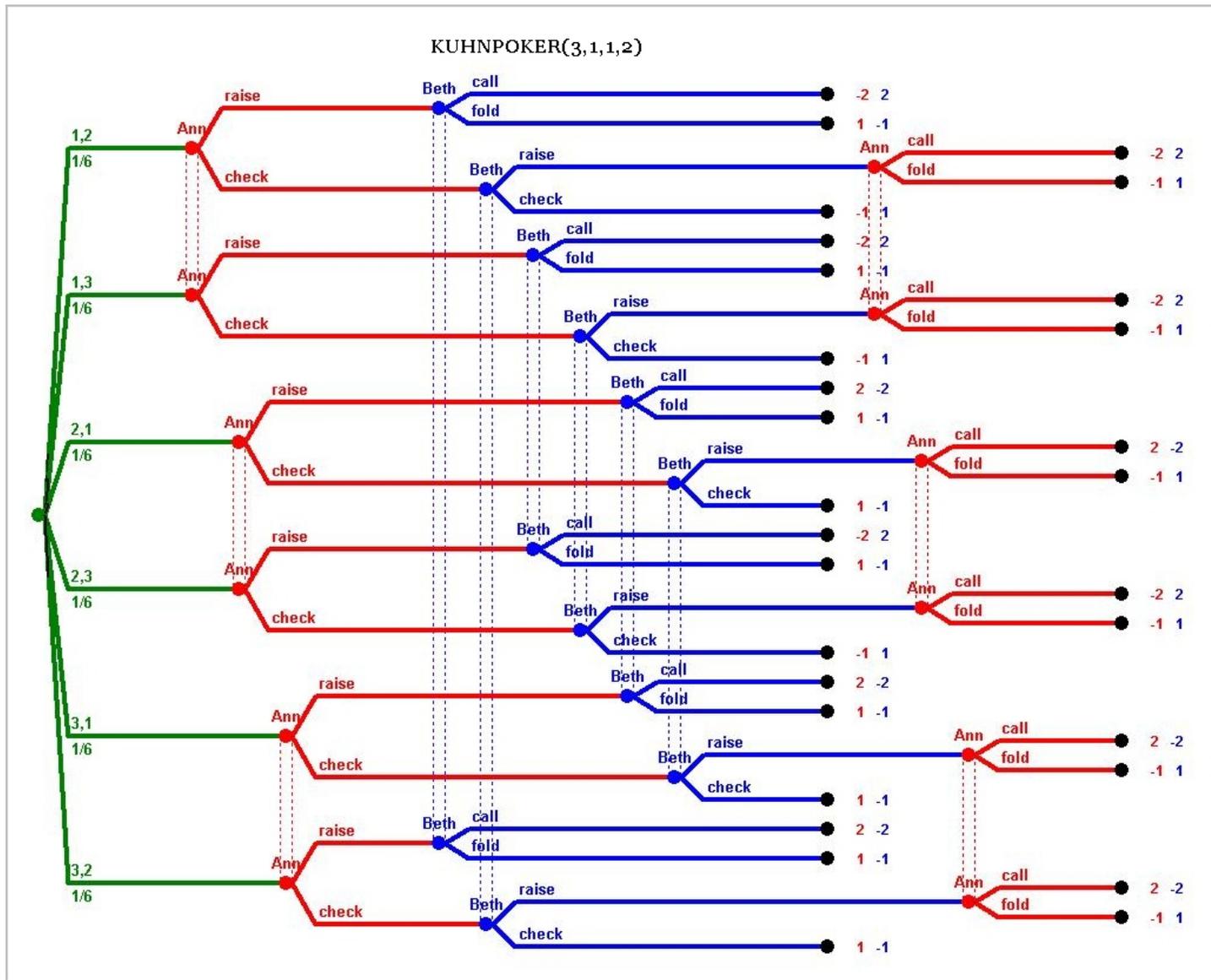
### Strategy from Cumulative Regret 2 Part 2

After these regrets are accumulated to the regret sum, what is the probability that player 2 will play B on the next iteration? \_\_\_\_\_

# Kuhn Poker Extensive Form

See [https://en.wikipedia.org/wiki/Kuhn\\_poker](https://en.wikipedia.org/wiki/Kuhn_poker) for details concerning Kuhn Poker.

The extensive form game tree for Kuhn Poker is given below:



## Kuhn Poker Player 1 Information Sets

How many information sets does player 1 have in Kuhn Poker? \_\_\_\_\_

## Kuhn Poker Player 2 Information Sets

How many information sets does player 2 have in Kuhn Poker? \_\_\_\_\_

## Counterfactual Regret Minimization

For CFR on a player 2 information set node, assume that realization weights are 0.250 and 0.100 for players 1 and 2, respectively. Further assume that, for that information set node, cumulative regrets are -300, 700, and 300 for actions a1, a2, and a3, respectively. If recursive calls to CFR for each of these actions yield action value 120, -200, and -240, respectively, what will be the new cumulative regrets for actions a1, a2, and a3, respectively? (Show your work to allow for partial credit in the event your final answer is incorrect.)

## Kuhn Poker Solutions

Run KuhnTrainer.java several times, observing the game value, which information set strategies change, which do not, and answer the following questions about optimal Kuhn Poker play.

### Kuhn Poker Game Value

The game value for player 1 is equal to a simple fraction that can be expressed as 1 divided which positive or negative integer? \_\_\_\_\_

### Kuhn Poker Advantage

For this zero-sum game, the first player game value implies that:

- There is a first player advantage.
- The game is fair, i.e. there is no first player advantage/disadvantage.
- There is a first player disadvantage.

### Kuhn Poker Player 1 Initial Betting with 1 Relative to Initial Betting with 3

For optimal play of the game of Kuhn Poker, let alpha and gamma be the probabilities that Player 1 bets immediately after being dealt a 1 and 3, respectively. (Recall that 3 is the highest card.) There are infinite Nash equilibria for Kuhn Poker. Indeed gamma can be any probability. However, given gamma, then alpha must be equal to gamma divided by which constant integer? \_\_\_\_\_

### Kuhn Poker Player 1 Second-Round Betting with 2 Relative to Initial Betting with 1

Let beta be the probability that Player 1 bets with a 2 on the second round (after a pass/check and a bet/raise). Given alpha, beta is always greater than alpha by a constant term. That constant is equal to a simple fraction that can be expressed as 1 divided which positive integer? \_\_\_\_\_

### Kuhn Poker Player 2 Bet with 1 Against a Pass

Player 2 only has a single optimal strategy. If player 2 is dealt a 1 and player 1 initially passes, player 2 will bet with a probability equal to a simple fraction that can be expressed as 1 divided which positive integer? \_\_\_\_\_

### Kuhn Poker Player 2 Bet with 2 Against a Bet

If player 2 is dealt a 2 and player 1 initially bets, player 2 will bet with a probability equal to a simple fraction that can be expressed as 1 divided which positive integer? \_\_\_\_\_