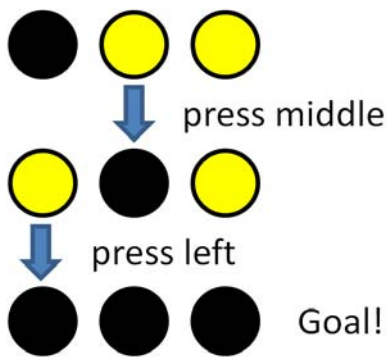
Games and Computation Homework #2: Game Representations

Answer these questions within the HW #2 Moodle quiz.

Lights Out in One Dimension

Imagine a row of *n* translucent buttons with an internal LED light, each of which may be on or off (i.e. lit or unlit). When a light is toggled, it changes to the opposite state: from on to off, or from off to on. When pressed, each button has the same behavior: that button and the button to its immediate left are both toggled. In the case of the leftmost button, both it and the rightmost button are toggled. (You can think of the button behaviors as wrapping around.) Here is an example of a 3 button row with the rightmost two buttons lit:



Pressing the middle button toggles it off and the left button on. Then pressing the left button toggles both it and the right button off. The goal of the puzzle is to take an initial state and press buttons so as to turn off all of the lights. Thus, the sequence of button presses above achieves the goal for this puzzle.

Select all of the following initial state descriptions for which NO sequence of button presses exists to achieve the goal:

- ☐ 123 lights are on and 234 lights are off.
- ☐ Half of 512 lights are on.
- ☐ 345 lights are on and 567 lights are off.
- ☐ 456 lights are on out of a total of 789 lights.

Corner-To-Corner Cabbie Can-Do?

- o First, watch the magic trick "How predictable are you?" on YouTube.
- o Next, try to figure out how the trick works.
- o Finally, read the explanation of the trick and use your understanding to answer the following question.

Consider a cab driver in the Northwest corner of a North-South-East-West *n*-by-*n* grid of intersections (*n* > 1). For which of the following conditions could the cab driver end up in the Southeast corner of the grid?

- ☐ The cab driver travels 6 * *n* times between intersections.
- ☐ The cab driver travels (100 * n) – 1 times between intersections.
- ☐ The cab driver travels n + 2 times between intersections and n > 42.
- ☐ The cab driver travels (42 * n) + 2 times between intersections and n > 42.

Five Free Tetrominoes Tiling Rectangles?

These are the free tetrominoes:

-  "I": four squares in a straight line.
-  "O": four squares in a 2×2 square.
-  "Z": two stacked horizontal dominoes with the top one offset to the left.
-  "T": a row of three squares with one added below the center.
-  "L": a row of three blocks with one added below the left side.

If the five free tetrominoes (I, O, Z, T, and L) can be flipped and or rotated, can one of each be tiled (i.e. fit together without overlapping) in the shape of a rectangle? If so, which rectangle size(s)? If not, why not? Hint: This is related to the mutilated checkerboard problem. (essay question)

Domineering Minimax Game Tree

The game of Domineering is played on an *m*-by-*n* grid (i.e. *m* horizontal rows, *n* vertical columns) where on each turn a player covers two adjacent empty squares as by placing a domino. The first player must only cover two vertically adjacent squares. The second player must only cover two horizontally adjacent squares. A player loses (and the opponent wins) when the player cannot make a legal play.

Construct the full minimax game tree for the 2x3 (i.e. 2 row, 3 column) game of Domineering. Then answer the following questions:

Who wins the game with perfect play, the first or second player? (answer "first" or "second") _______________

How many nodes are there in the full game tree? _______________

In game-tree search, a "transposition table" is used to reduce redundant search by allowing an algorithm to see if an identical or equivalent (e.g. symmetric) position has already been analyzed. If we were to run a complete search through the 2x3 Domineering game tree, how many unique states would the transposition table contain? Hint: Horizontally- and vertically-mirrored images of the board are equivalent game-theoretically. _______________