Games and Computation Homework #3:
Game Trees/Graphs and Minimax

Answer these questions within the HW #3 Moodle quiz:

Miseré Nim
Whereas Normal Play Nim has the object of being the last player to play (i.e. take the last piece(s)), Miseré Nim has the object of not being the last player to play. In other words, you want the opponent to be the one that takes the last piece. (Play online: http://www.archimedes-lab.org/game_nim/play_nim_game.html)

Let us consider the three pile game where we begin with one pile has 1 piece and two piles have 2 pieces each. Further, let us represent (i.e. denote) this game situation by sorting the piles from greatest to least number of pieces and listing those sizes. For this simple game, the initial state would be represented as “221”.

Create a game graph of Miseré Nim and evaluate it using Minimax. Note: By representing the game as sorted pile numbers, you should be able to avoid much game graph redundancy.

If a win for the maximizing first player (“max”) is +1, and a win for the minimizing second player (“min”) is -1, what is the game value of the initial game state? __________

What is max’s optimal move in that state? Please represent it as the resulting successor state in the three digit representation above with no spaces. __________

What state should min then play to if max plays to 211? Please represent it as the resulting successor state in the three digit representation above with no spaces. __________

Chomp with numeric non-increasing rows representation
Represent any state in a 3x4 Chomp game as a sequence of unchomped row lengths from the top row with the poisoned square (greatest) to the bottom row (least). So the initial state is 444 and can play to 443, 442, 441, 440, 433, 422, 411, 400, 333, 222, 111, and 000. How many board states exist for the 3x4 game (ignoring who is the current player, and ignoring diagonal symmetries (e.g. only represent 111 or 300 but not both))? That is, how many different non-increasing 3-number sequences exist for this 3x4 game? _____

How many state 3-number sequences are there where play against a perfect opponent results in a loss? _____

A state is a winning state if the opponent has just chomped the poisoned square (000) or one can play to a losing state. A state is a losing state if one can only play to winning states. Represent the following optimal plays by giving a non-increasing row description of the resulting board state:

What is the winning play from 444? _____
What is the winning play from 440? _____
What is the winning play from 330? _____
What is the winning play from 411? _____
What is the winning play from 211? _____

What is the winning play from 222? _____

What is the winning play from 400? _____

What general winning play rule can be generalized from winning plays from 411 and 211? The rule should apply to situations on any size board. (essay question)

What general winning play rule can be generalized from winning plays from 440, 330, and 222? The rule should apply to situations on any size board. (essay question)

**3-Player Evaluation**

Consider the following 3-player game tree. Nodes labelled with numbers represent choice nodes where the given number player is making the decision from among two plays to child nodes. Nodes labelled with letters represent terminal nodes with a payoff vector containing utilities for each player in order of player number.

![Game Tree](image)

For the following questions, you will assume the following values for the terminal nodes:

A: (9, 14, 5)
B: (6, 3, 21)
C: (17, 19, 20)
D: (13, 15, 16)
E: (8, 22, 23)
F: (2, 12, 11)
G: (1, 4, 18)
H: (7, 10, 3)

If each player is choosing a play that maximizes its own utility in the payout vector ("max_"), which letter node will such play reach? _____

If player 1 is maximizing player 1’s utility, and the other players are minimizing player 1’s utility ("against_"), which letter node will such play reach? _____

If player 2 is maximizing player 2’s utility, and the other players are minimizing player 2’s utility ("against_"), which letter node will such play reach? _____

If player 3 is maximizing player 3’s utility, and the other players are minimizing player 3’s utility ("against_"), which letter node will such play reach? _____