

# Games and Computation Homework #8: Games of Chance and Computations of Pig

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Answer these questions within the HW #8 Moodle quiz:

## Expected Turn Total

What is the expected turn total immediately before a Pig (1) roll? Hint: We computed the expected number of heads flipped before a tail in class. Use the same approach with either algebraic solution or value iteration. \_\_\_\_\_

## Maximizing Turn Score with 6 Pig

In class, we discussed why "Hold at 20" optimizes score gain per turn for the game of Pig. If Pig were modified so that a roll of 1 added 1 to the turn total and that a roll of 6 was the "Pig" roll that ended the turn with no score, at which turn total (and above) would you hold to maximize score gain? \_\_\_\_\_

## Maximizing Turn Score with Two Dice

In class, we discussed why "Hold at 20" optimizes score gain per turn for the game of Pig. If Pig were modified so that two dice were rolled at a time (with the same consequences), at which turn total (and above) would you hold to maximize score gain? Note: Although the expected turn total gain of a roll is not an integer, your turn total will always be an integer, so your answer should be expressed as an integer. \_\_\_\_\_

## Pig-Hog Hybrid

In Hog, one gets a single roll on one's turn of as many dice as desired and these are scored in the same way as if they'd been rolled sequentially in any order (ignoring any rolls after the first 1 is encountered). Now imagine a Pig-Hog hybrid game where one can roll many times in a turn, and may roll as many dice per roll (Hog-style) as one wishes. True or false: There exists at least one situation for which the optimal player must choose to roll more than one die at a time.

- True
- False

## Squared Sum of Two Dice

Imagine a simple game in which a player scores the square of the sum of two dice. What would be the expected score of such a two dice roll? Hint: There are 11 possible outcomes. If one can compute the outcomes (e.g.  $7 * 7 = 49$ ) and the probability of each outcome (e.g. 36 rolls with 6 ways to roll a total of 7: 1-6, 2-5, 3-4, 4-3, 5-2, 6-1, so  $6/36 = 1/6$  is the probability of rolling a total of 7), this is just the evaluation of a single chance node. Round your answer to two places after the decimal point. \_\_\_\_\_

## Approach 5 Optimal Hold Value

Using the techniques from class, calculate and enter the optimal first player hold value for Approach 5. \_\_\_\_\_

## Approach 5 Optimal Win Probability

Using the techniques from class, calculate and enter the first player win probability when using the optimal hold value. Round your answer to two places after the decimal point. \_\_\_\_\_

## Bellman's Optimality Equations Notation

Match the following notational symbols with their interpretation for Bellman's Optimality Equations.

Notational Symbols (in alphabetical order)

- $a$  in  $A(s)$
- $\gamma$
- $P^a_{s,s'}$
- $R^a_{s,s'}$
- $s, s'$
- $V(s)$

Interpretations of symbols (in alphabetical order):

- a legal action in state  $s$
- a state, a minimal set of variables relevant to optimal decisions of the Markov Decision Process (MDP)
- the discount factor for discounting the value of future rewards
- the expected future value from state  $s$
- the immediate reward for taking action  $a$  and transitioning from state  $s$  to state  $s'$
- the probability of transitioning to state  $s'$  when taking action  $a$  in state  $s$

## Bellman's Optimality Equations and Pig

Applying Bellman's Optimality Equations to Pig with our turn-changing modifications, let us suppose that we have all zero immediate rewards. Given this fact, if we wish to have the expected future value of state  $s$  be the probability of winning, then what must that value be for all states where  $i + k$  are greater than or equal to the goal score?

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## Bellman's Optimality Equations and Pig 2

Applying Bellman's Optimality Equations to Pig with our turn-changing modifications, let us suppose that we have all zero immediate rewards. Given this fact, if we wish to have the expected future value of state  $s$  be the probability of winning, then what value must we assign to the discount factor? \_\_\_\_\_