



Dudo: An Introduction to the Incan Bluffing Dice Game

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Dudo

- “According to legend, King Atahualpa of the Incas taught this to the Spanish conquistador Pizarro more than 400 years ago...” (Mohr, 1997)
- Origin generally believed to be 15th c. Inca
- Variants long popular in many Latin American countries







































Beyond Latin America

- Many variants/names (Liar's Dice, Perudo, Bluff, Call My Bluff, Cacho, Cachito)
- Internationally popular
 - BoardGameGeek.com rank 236/49160 (top ½%!), 11/2/10

Dudo Overview

- Bluffing dice game for 2+ players
 - Each player rolls 5 dice concealed under cup
 - Players make successively bolder claims about *all* dice rolled until player challenges
 - Loser of challenge loses dice
 - Last player with dice wins

Dudo Claims

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | |
| 1 |  | 2 |  | 2 |  | 2 |  | 2 |  | 2 |  |
| | 3 |  | 3 |  | 3 |  | 3 |  | 3 |  | |
| 2 |  | 4 |  | 4 |  | 4 |  | 4 |  | 4 |  |
| | 5 |  | 5 |  | 5 |  | 5 |  | 5 |  | |
| 3 |  | 6 |  | 6 |  | 6 |  | 6 |  | 6 |  |
| | 7 |  | 7 |  | 7 |  | 7 |  | 7 |  | |

Dudo Rules

- Players each roll and privately view 5 dice concealed under a cup. 1's are wild.
- Players make successively greater claims until one challenges the previous claim with “Dudo!” (Sp. “I doubt it!”), all reveal dice, and:
 - More/less than claimed? Challenger/claimant loses dice according to difference.
 - Claim exactly correct? Challenger loses 1 die.
- The next round begins with challenge winner.
- The last player with dice wins.

Computer Play

- Gather around lab computers. (Carefully fold down desks.)
- Commands:
 - `cp ~tneller/pub/java/dudo/*.java .`
 - `javac *.java`
 - `java DudoGame`
- Input claim format: “dudo” or “<num> <rank>” (e.g. “7 6”)
- What do you observe?

Simple Dudo Player

- SimpleDudoPlayer follows a simple policy based on roll probabilities.
- Make the strongest legal claim that is correct with probability $\geq \frac{1}{2}$.
- If no such claim exists, call “dudo”.

Calculating Claim Probabilities

- Example: What is the probability of exactly 2 of 5 dice being 6's?
 - 1's are wild, so the probability of a single die being a 6 is $1/3$.
 - 6^5 different 5 dice rolls (in sequence)
 - 5 choose 2 = $5! / (2! 3!) = 120 / (2 * 6) = 10$ different ways of choosing 2 from 5
 - Each occurrence has prob. $(1/3)^2(1-1/3)^3$
- Answer: $10 \times (1/3)^2(1-1/3)^3 = 80/243$

Calculating Claim Probabilities (cont.)

- Example: What is the probability of **at least 2** of 5 dice being 6's?
 - Exactly 0: $1 \times (1/3)^0(1-1/3)^5 = 32/243$
 - Exactly 1: $5 \times (1/3)^1(1-1/3)^4 = 80/243$
 - Exactly 2: $10 \times (1/3)^2(1-1/3)^3 = 80/243$
 - Exactly 3: $10 \times (1/3)^3(1-1/3)^2 = 40/243$
 - Exactly 4: $5 \times (1/3)^4(1-1/3)^1 = 10/243$
 - Exactly 5: $1 \times (1/3)^5(1-1/3)^0 = 1/243$
 - At least 2: $(80 + 40 + 10 + 1) / 243 = 131/243 \approx .539$

Choosing a Claim

- Example computation:

```
Total number of dice: 10  
Number of known values: 5  
Known values: 1 2 2 6 6
```

| | Count | | | | | | | | | |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Rank | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1: | 1.000 | 0.598 | 0.196 | 0.035 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2: | 1.000 | 1.000 | 1.000 | 0.868 | 0.539 | 0.210 | 0.045 | 0.004 | 0.000 | 0.000 |
| 3: | 1.000 | 0.868 | 0.539 | 0.210 | 0.045 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4: | 1.000 | 0.868 | 0.539 | 0.210 | 0.045 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5: | 1.000 | 0.868 | 0.539 | 0.210 | 0.045 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6: | 1.000 | 1.000 | 1.000 | 0.868 | 0.539 | 0.210 | 0.045 | 0.004 | 0.000 | 0.000 |

- SimpleDudoPlayer chooses the strongest claim that is correct with probability $\geq \frac{1}{2}$:
five 6's

Game Information

- Dudo is a game of information:
 - State information: What you hold, how many dice others have
 - Probabilistic information: Roll probabilities
 - “Tell” information: Behavior indicators
 - **Play information: Other player actions**
- Now that you know how SimpleDudoPlayer makes decisions, exploit that information.

Mixed Strategy

- Predictability \rightarrow Exploitability
- Are these also examples of games with optimal mixed strategy?
 - Tic Tac Toe
 - Rock Scissors Paper
 - Blackjack / Pig
 - Poker
 - Clue
- Game-theoretic techniques help us compute or approximate optimal mixed strategies

Conclusion

- Dudo is an excellent, ancient bluffing dice game.
- Simple application of probability and combinatorial mathematics yields a reasonable simple player.
- Predictability \rightarrow Exploitability
- Because player choices yield information, games of information often call for mixed strategies.

References

- Reiner Knizia. ***Dice Games Properly Explained***. Elliot Right-Way Books, Brighton Road, Lower Kingswood, Tadworth, Surrey, KT20 6TD UK, 1999
- Marilyn Simonds Mohr. ***The New Games Treasury – More Than 500 Indoor and Outdoor Favorites with Strategies, Rules and Traditions***, Houghton Mifflin Co., Boston, 1997
- Dudo rules online:
<http://cs.gettysburg.edu/~tneller/games/rules/dudo.pdf>