



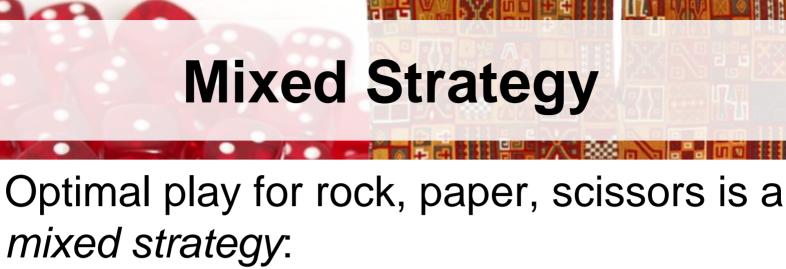
- Many variants/names (Liar's Dice, Perudo, Bluff, Call My Bluff, Cacho, Cachito)
- Internationally popular
  - BoardGameGeek.com rank 173/4268 w/ 30+ ratings, 30K+ games (top 4% of 14% most rated), 2/4/08



- Bluffing dice game for 2+ players
  - Each player rolls 5 dice concealed under cup
  - Players make successively bolder claims about all dice rolled until player challenges
  - Loser of challenge loses dice
  - Last player with dice wins



- Players each simultaneously reveal rock, paper, or scissors
  - Rock beats scissors
  - Scissors beats paper
  - Paper beats rock



 A strategy which chooses proportionally between actions at random

# Optimal Play in Rock, Paper, Scissors

- Choosing one action consistently
  - Predictable
- Optimal Play:
  - Throw rock 1/3 of the time
  - Throw paper 1/3 of the time
  - Throw scissors 1/3 of the time

### **Systems of Equations**

$$5x + y = 13$$
$$3x - y = 3$$

• Solution: x = 2, y = 3



(equations / inequalities) of unknowns

# Rock, Paper, Scissors Linear Program

```
\max_{x,q} -q_2
                                  -q_1 + q_2 \leq 0
                 -x_3 +x_4 -q_1 +q_2 \le 0
                        -\mathbf{x}_4 \quad -\mathbf{q}_1 \quad +\mathbf{q}_2 \leq 0
                                           +q_2 \leq 0
                                  -\mathbf{q}_1
         -\mathbf{x}_2 + \mathbf{x}_3
                                  -q_1 + q_2 = 0
+\mathbf{X}_1
-x_1 + x_2 + x_3 + x_4 - q_1 + q_2 = 1
x_1, x_2, x_3, x_4 \ge 0
```

#### Linear Program for Dudo

- Rock, paper, scissors
  - 6 relations and 6 unknowns to solve optimal play for both player 1 and player 2
- Endgame Dudo: 2 players, 1 die each
  - 36,866 relations and 36,872 unknowns to solve optimal play for player 1
  - 36,872 relations and 36,866 unknowns to solve optimal play for player 2
  - Matrices with 1,359,323,152 entries each.

#### **Endgame Dudo Claims**

- 1's are wild (counted as any # incl. 1)
- Claims are ordered left-to-right, top-tobottom (reading order)
- 1x2 is least claim; 2x1's is greatest claim

		1	•	1	••	1	••	1	1	
1	•	2	•	2	••	2	• •	2	2	
2	•									

#### **Endgame Dudo Rules**

- 2 players each roll and privately view 1 die concealed under a cup. 1's are wild.
- Players make successively greater claims until one challenges the previous claim with "Dudo!" (Sp. "I doubt it!"), both reveal dice, and:
  - Less than claimed? Challenger wins!
  - More/equal? Claimant wins!

### **Endgame Dudo Claims**

Let's play!

		1 .	1 ••	1 .	1	1 🔡
1	•	2 .	2 •••	2	2 ::	2 🔡
2	•					

# Sharing vs. Concealing Information

- Making a true claim too consistently shares information to other players about which dice you are holding giving them an advantage
- If you bluff too often, others will call your bluff often, causing you to lose often
- Mixed strategy

### An Example of Optimal Play

- First player rolls a •.
  - Claim 1 : 18/43rds of the time
  - Claim 1 : 16/43rds of the time
  - Claim 1 : 9/43rds of the time
- This is just one initial possible situation
- We computed optimal responses to all of the many possible rolls and claim sequences

## **Dudo Claims**

		1	•	1	••	1	••	1		1	
1	•	2	•	2	••	2		2	<b>::</b>	2	•
		3	•	3	••	3		3	<b>::</b>	3	•
2	•	4	•	4	••	4	••	4	<b>::</b>	4	
		5	•	5	••	5	••	5	<b>::</b>	5	
3	•	6	••	6	••	6	••	6	::	6	
<b>1770   37</b>		7		7		7		7		7	•••



- Players each roll and privately view 5 dice concealed under a cup. 1's are wild.
- Players make successively greater claims until one challenges the previous claim with "Dudo!" (Sp. "I doubt it!"), all reveal dice, and:
  - More/less than claimed? Challenger/claimant loses dice according to difference.
  - Claim exactly correct? Each player loses 1 die.
- The next round begins with challenge winner.
- The last player with dice wins.

#### References

- Reiner Knizia. *Dice Games Properly Explained*. Elliot Right-Way Books, Brighton Road, Lower Kingswood, Tadworth, Surrey, KT20 6TD UK, 1999
- Merilyn Simonds Mohr. The New Games
   Treasury More Than 500 Indoor and Outdoor
   Favorites with Strategies, Rules and Traditions,
   Houghton Mifflin Co., Boston, 1997
- Dudo rules online: http://cs.gettysburg.edu/~tneller/games/rules/dudo.doc