

# First Player's Cannot-Lose Strategies for Cylinder-Infinite-Connect-Four with Widths 2 and 6

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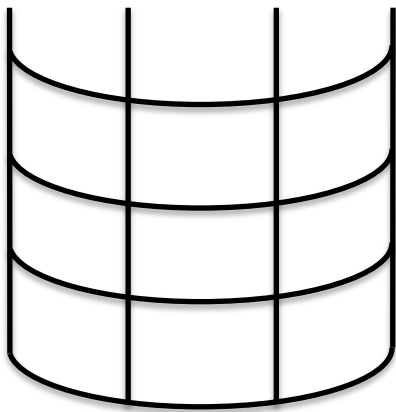
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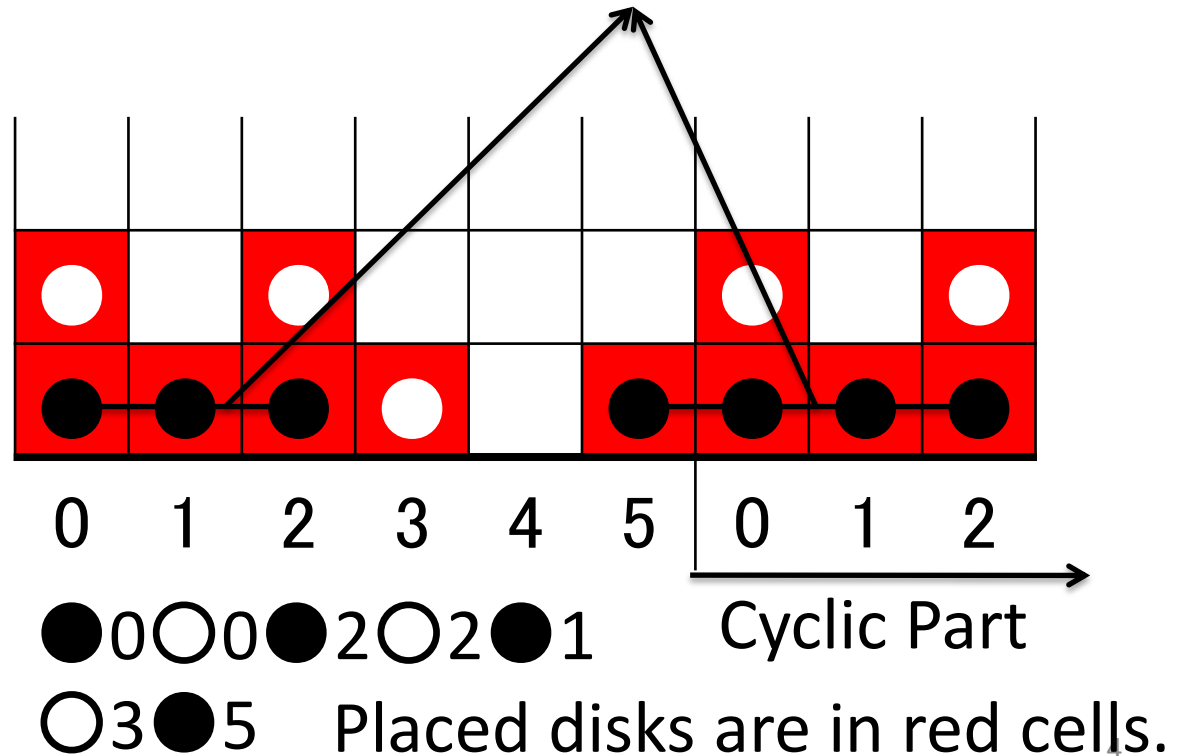
# About Cylinder-Infinite-Connect-Four

# Rules of Cylinder-Infinite-Connect-Four

- The board is infinite in height and cyclical in width.
- The first player who achieves a Connect4 wins.

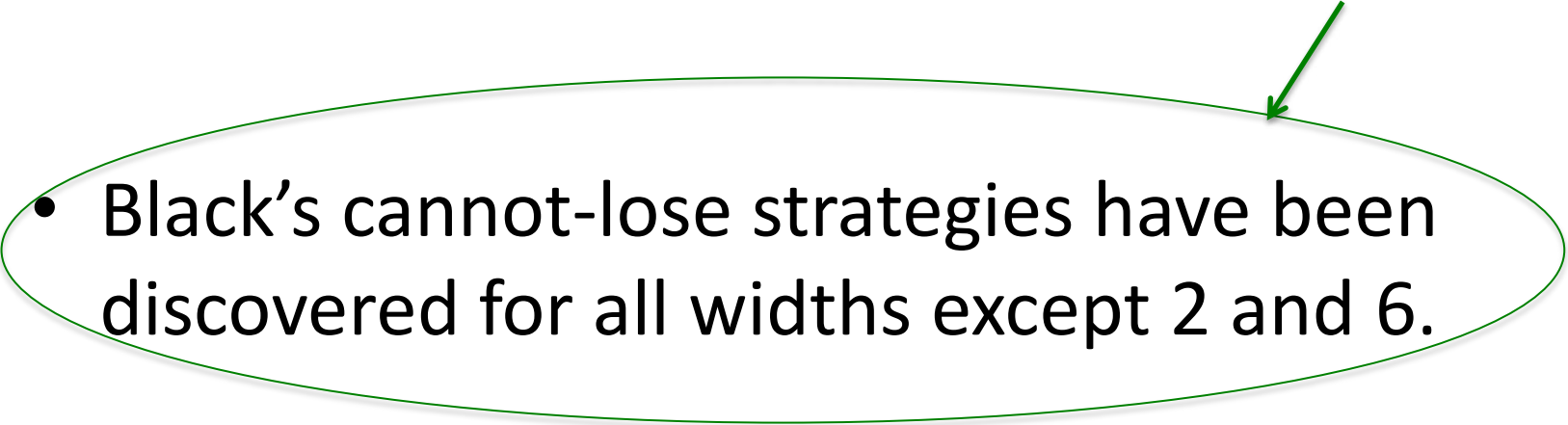


Board



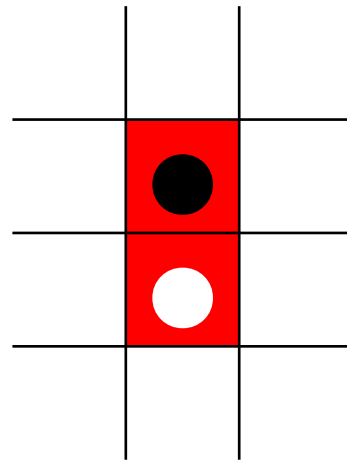
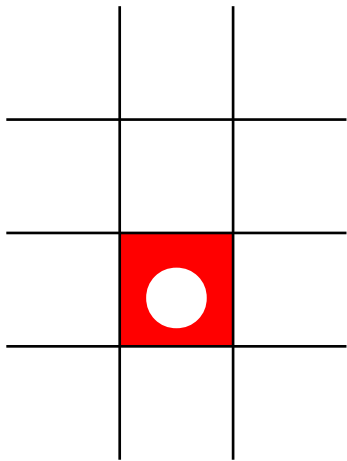
# Previous Work

Solved at this time!

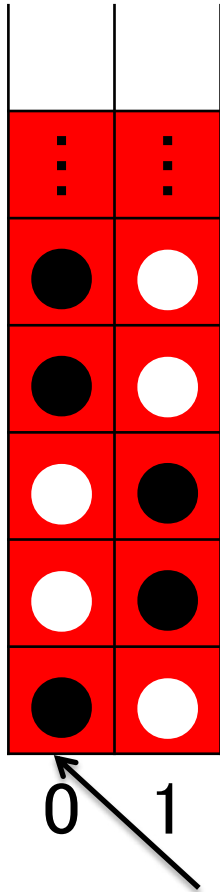
- 
- Black's cannot-lose strategies have been discovered for all widths except 2 and 6.
  - White's cannot-lose strategies have been discovered for all widths except 6 and 11.

# First Player's Cannot-Lose Strategy in Cylinder-Infinite-Connect-Four for Width 2

# Follow-up



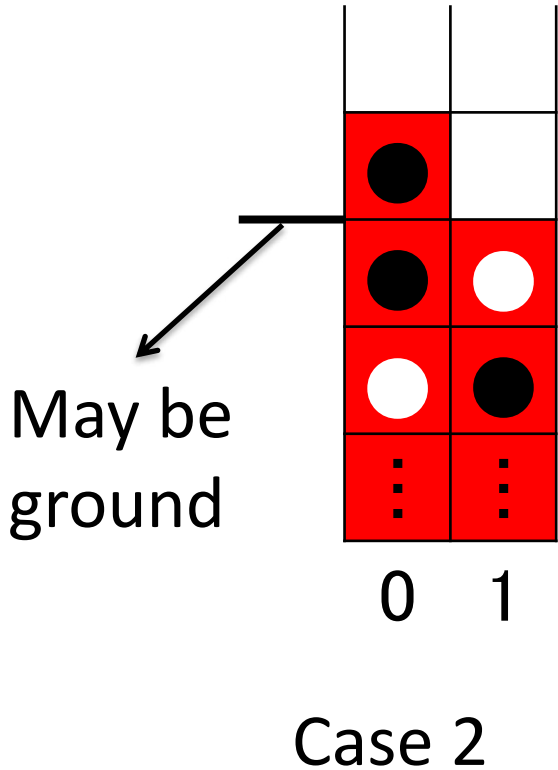
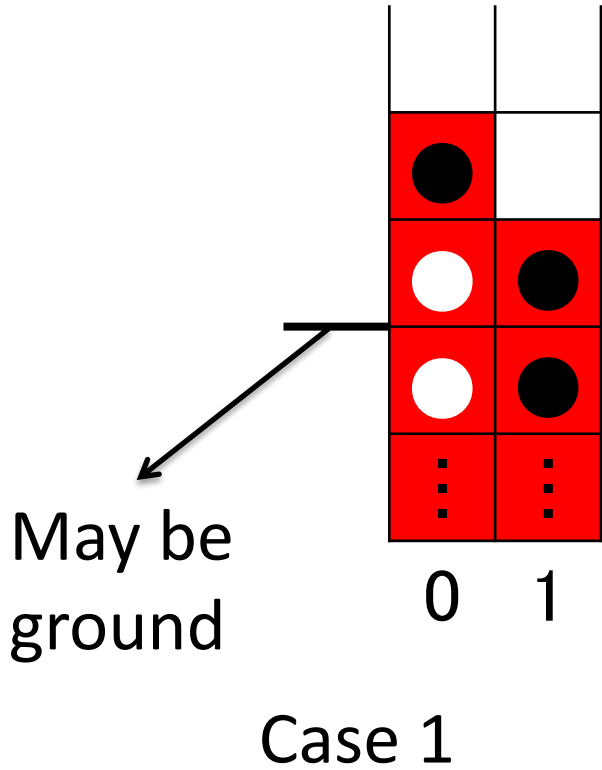
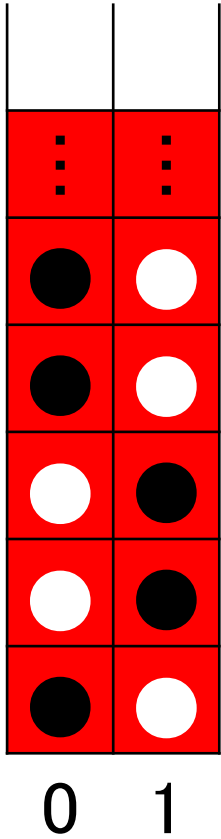
# As Long As White Does Not Play Follow-up



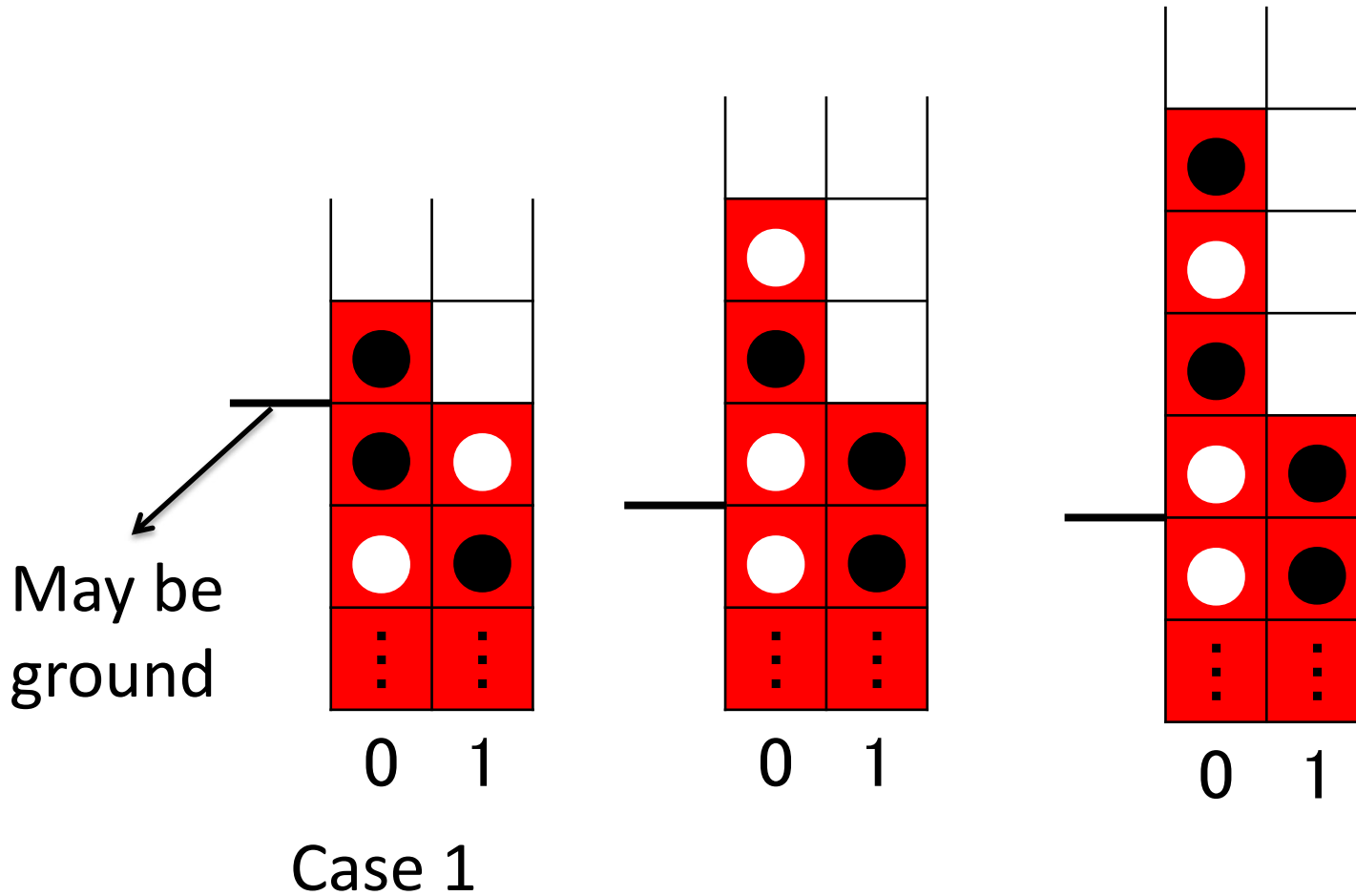
Black alternates between follow-up and non-follow-up plays, starting with follow-up.



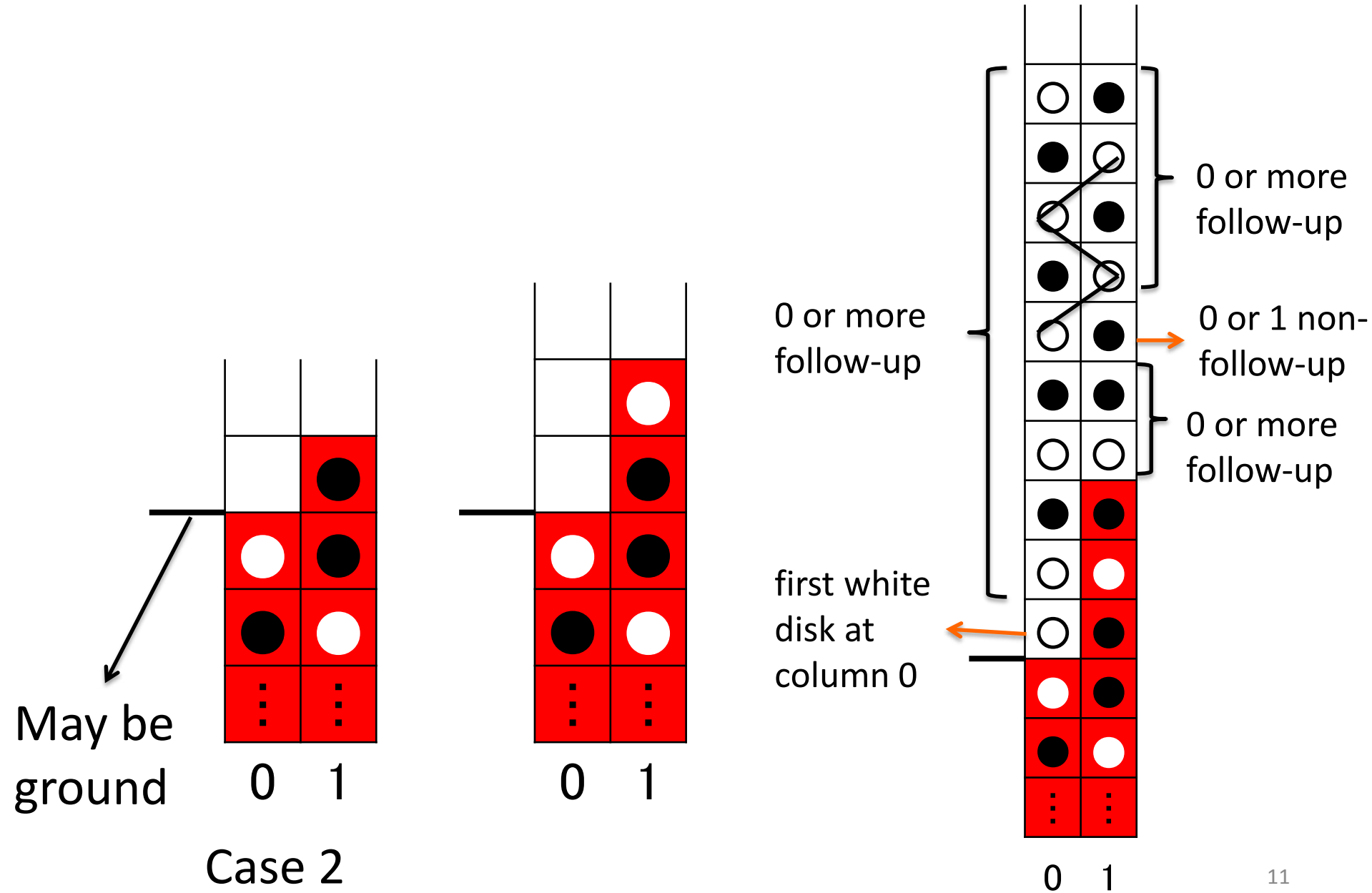
# 2 Cases Where White Plays Follow-up



# White Plays Follow-up After Case 1

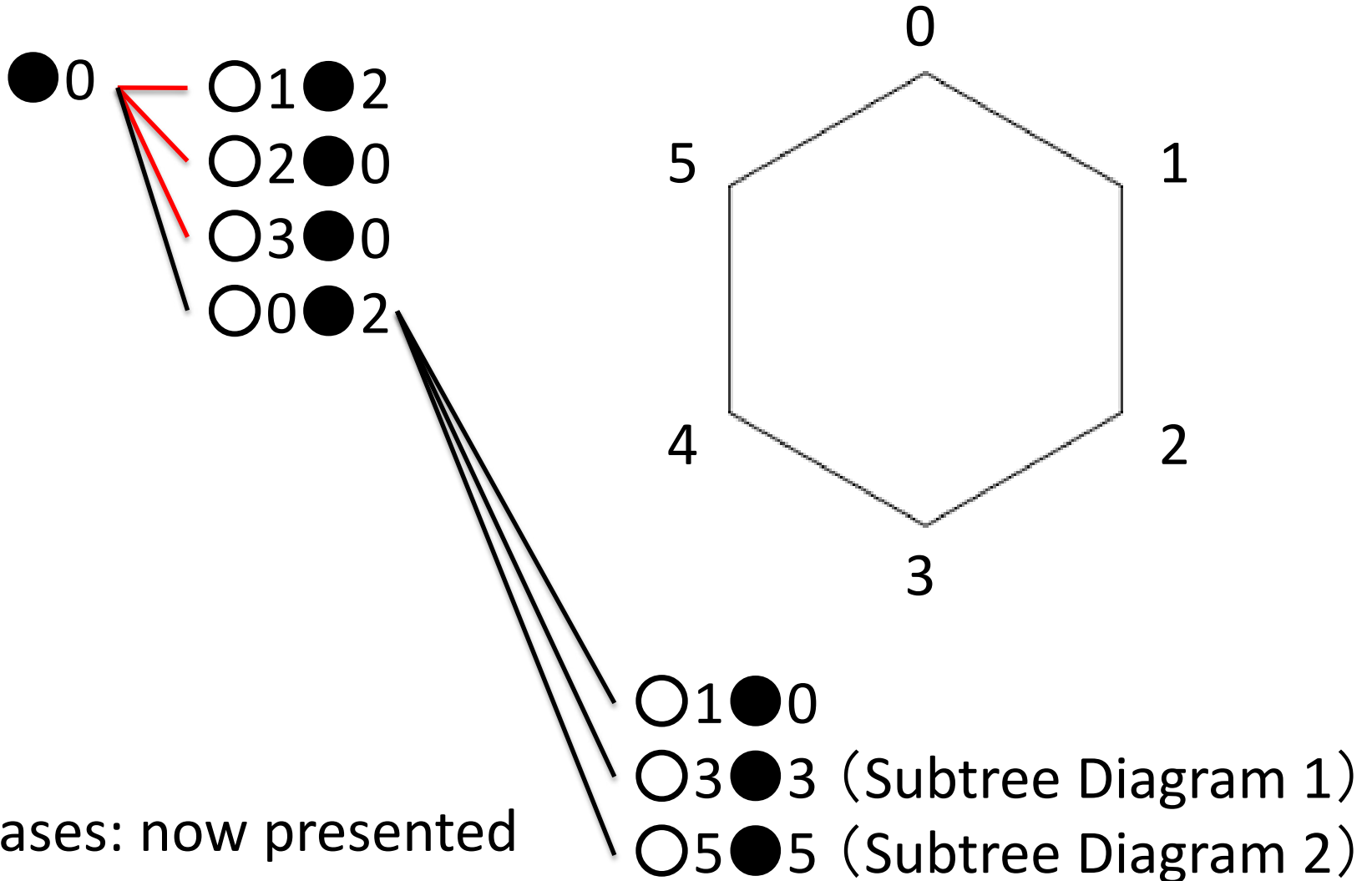


# White Plays Follow-up After Case 2



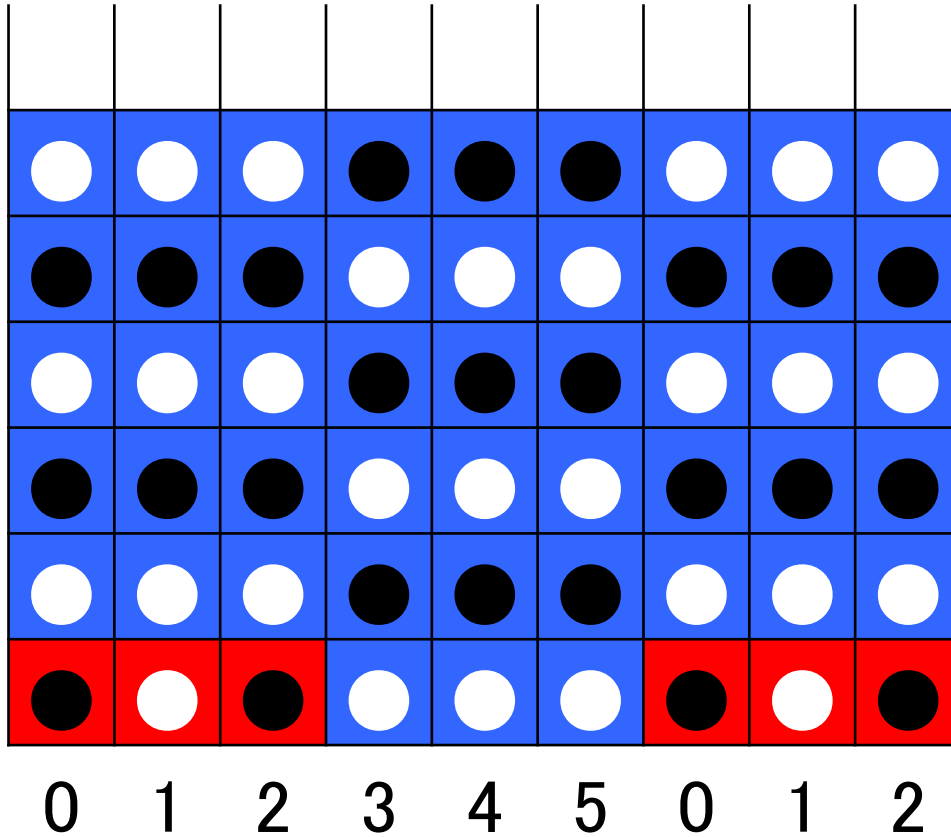
# First Player's Cannot-Lose Strategy in Cylinder-Infinite-Connect-Four for Width 6

# Tree Diagram



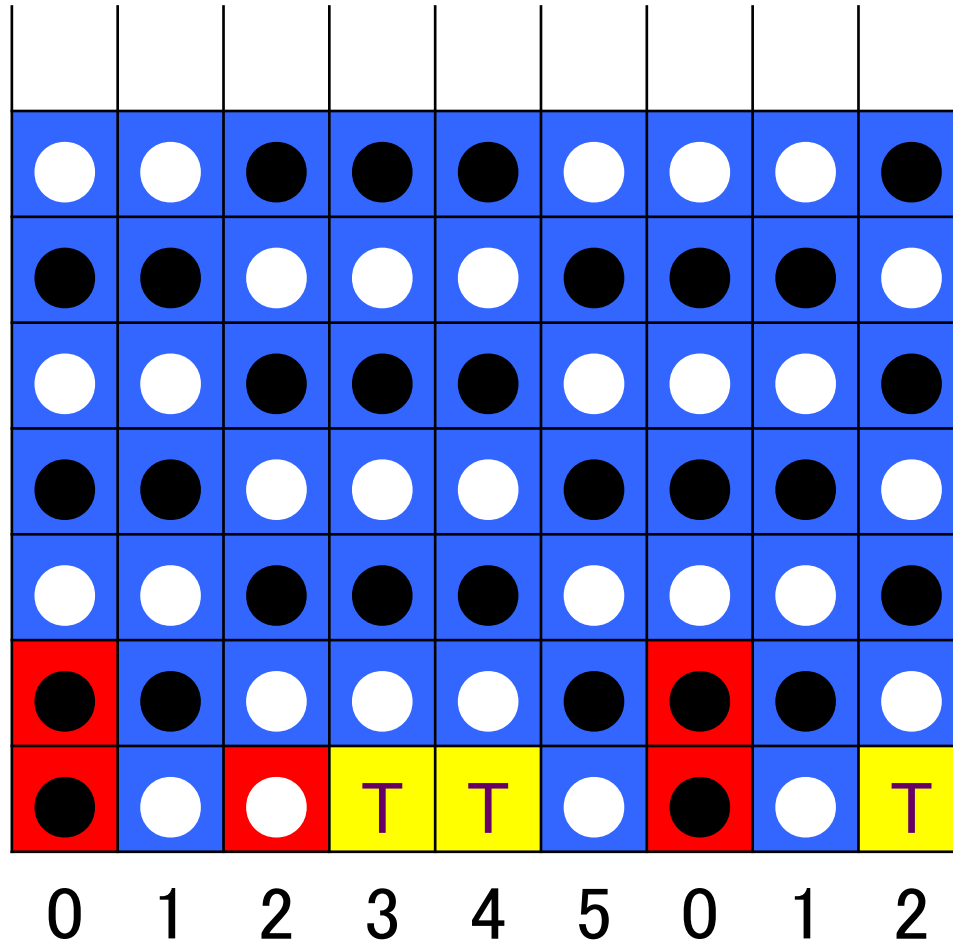
Red cases: now presented  
Black cases: non-covered

● 0 ○ 1 ● 2



Black plays only follow-up at blue cells.

● 0 ○ 2 ● 0



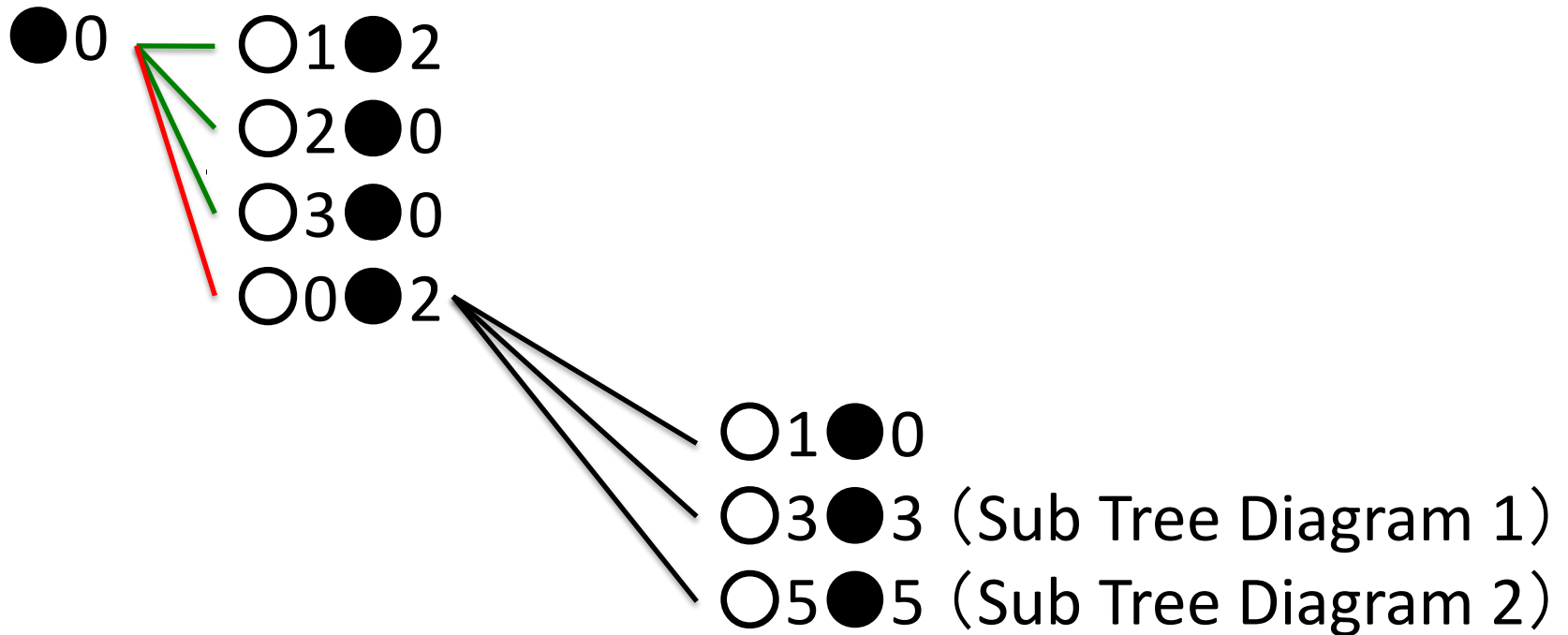
T: Tile (Yellow Cells)

●0○3●0

○	○	●	●	●	○	○	○	●
●	●	○	○	○	●	●	●	○
○	○	●	●	●	○	○	○	●
●	●	○	○	○	●	●	●	○
○	○	●	●	●	○	○	○	●
●	●	○	○	○	●	●	●	○
●	○	T	○	T	○	●	○	T
0	1	2	3	4	5	0	1	2

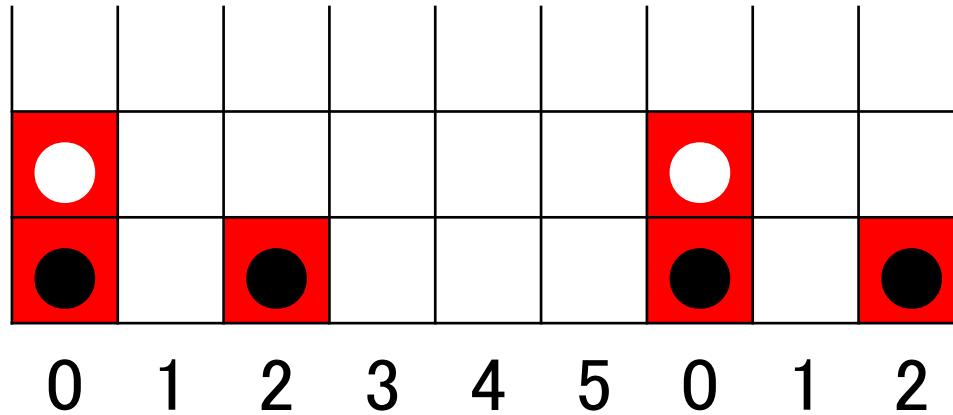


# Tree Diagram

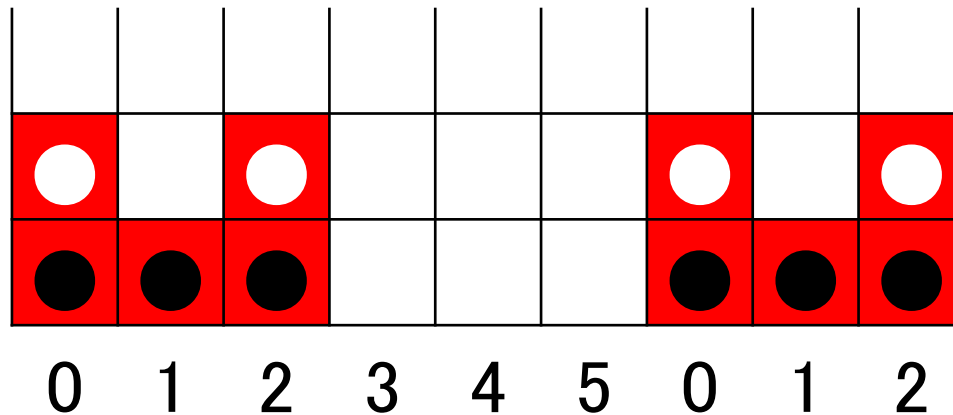


Green cases: covered

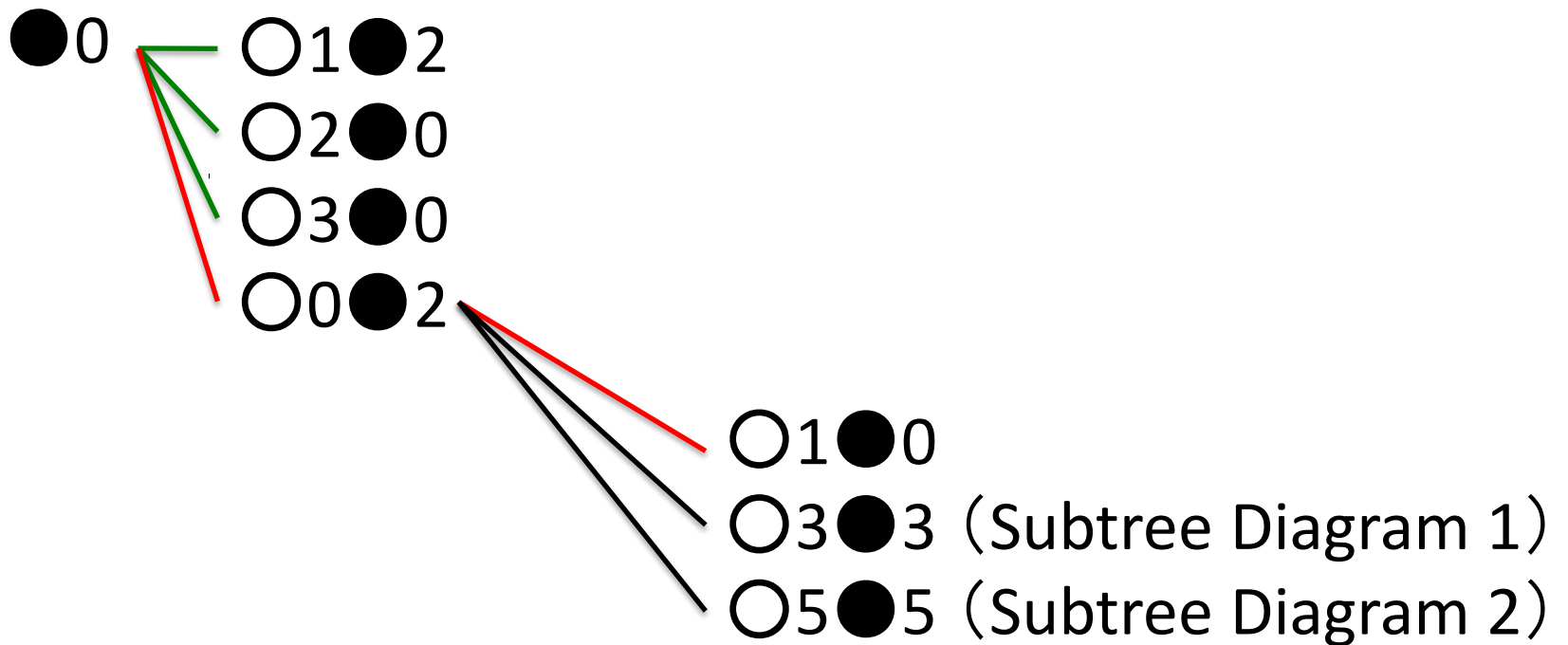
After ●0○0●2, White Must Play  
in Column 1, 3, or 5



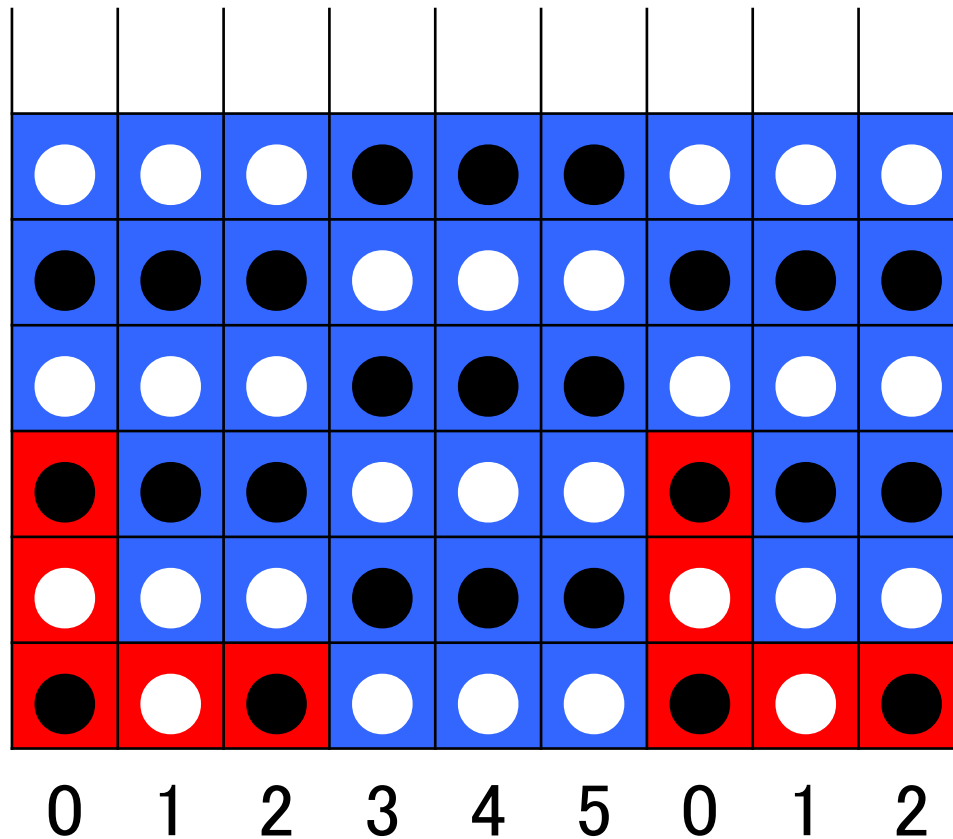
Otherwise, Black plays in column 1 and can achieve a  
Connect4.



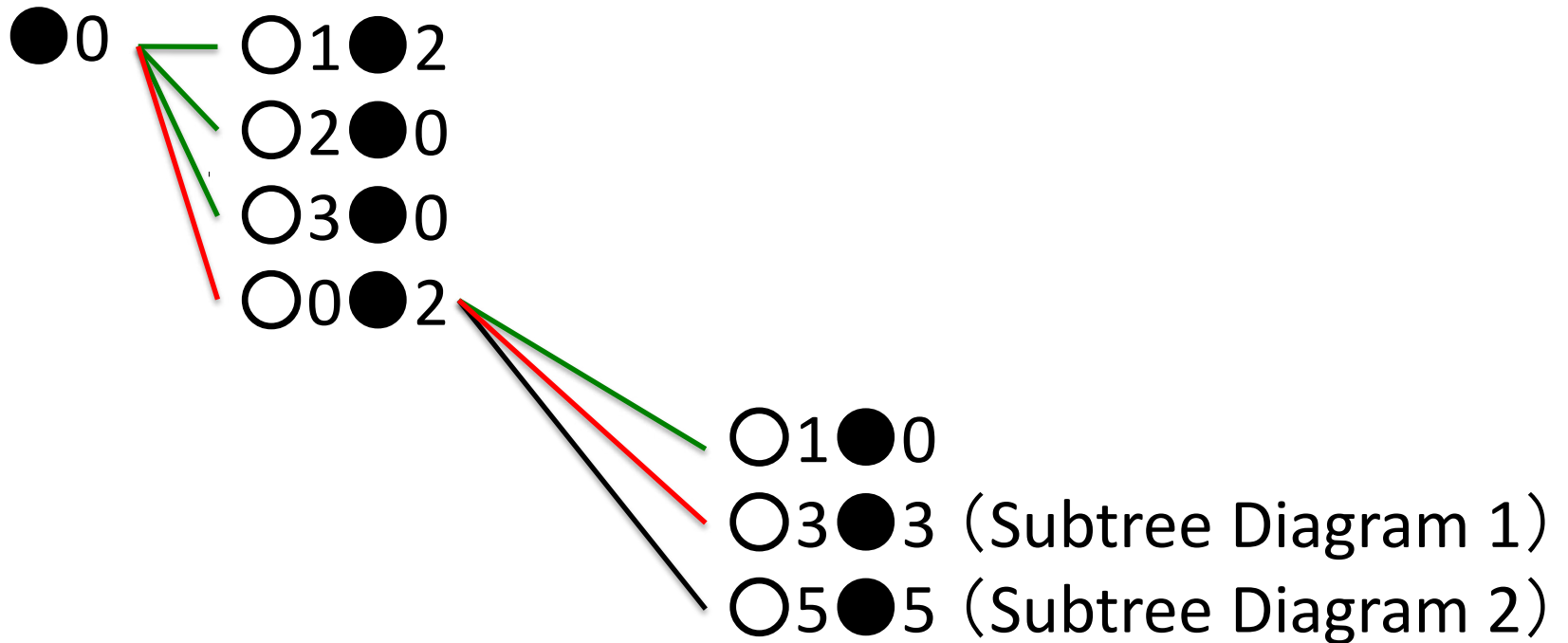
# Tree Diagram



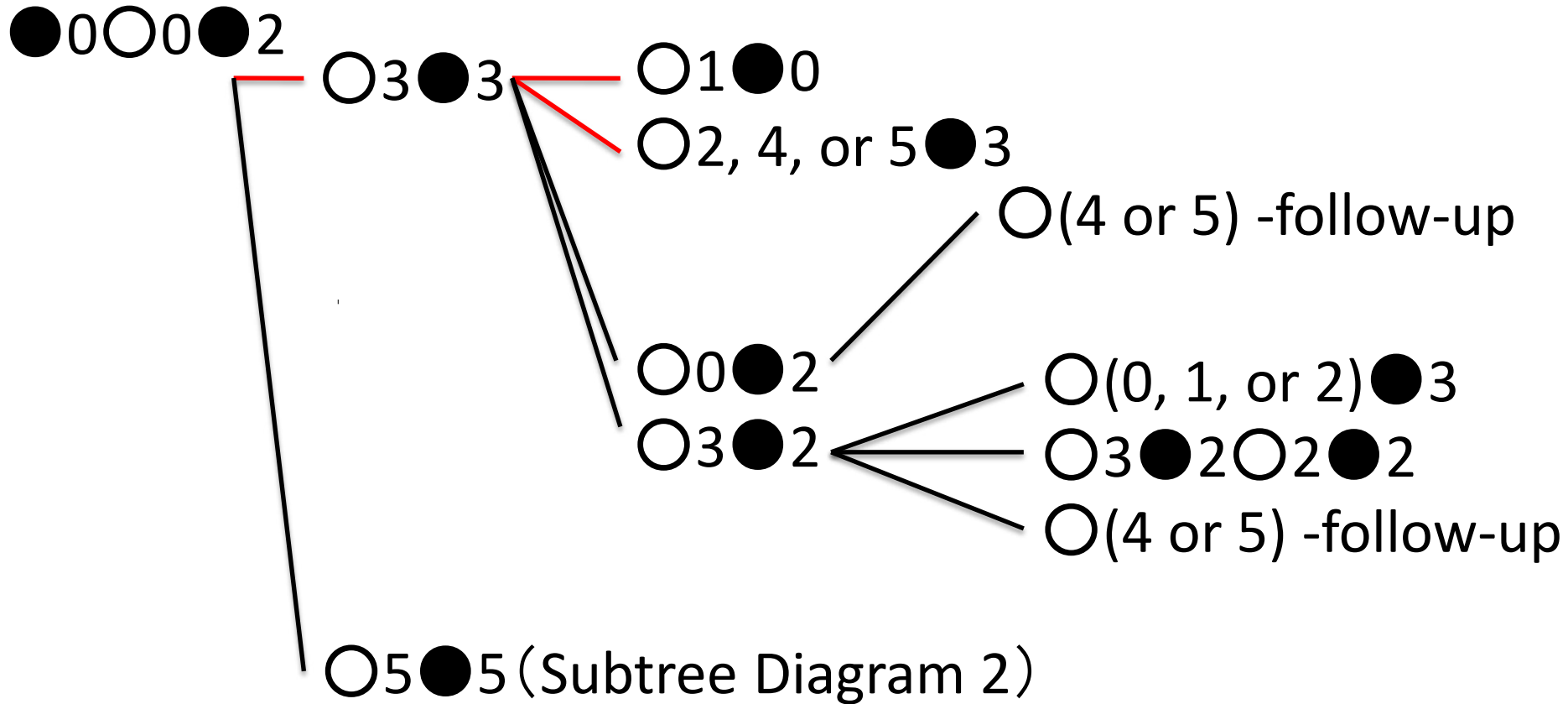
● 0 ○ 0 ● 2 ○ 1 ● 0



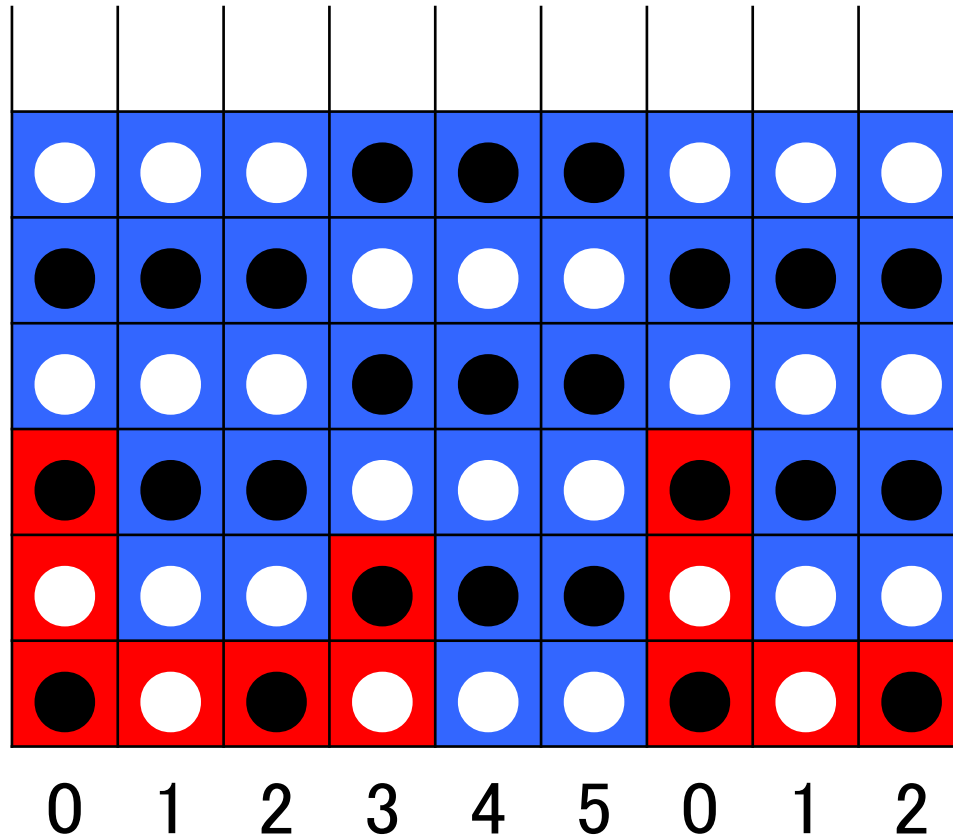
# Tree Diagram



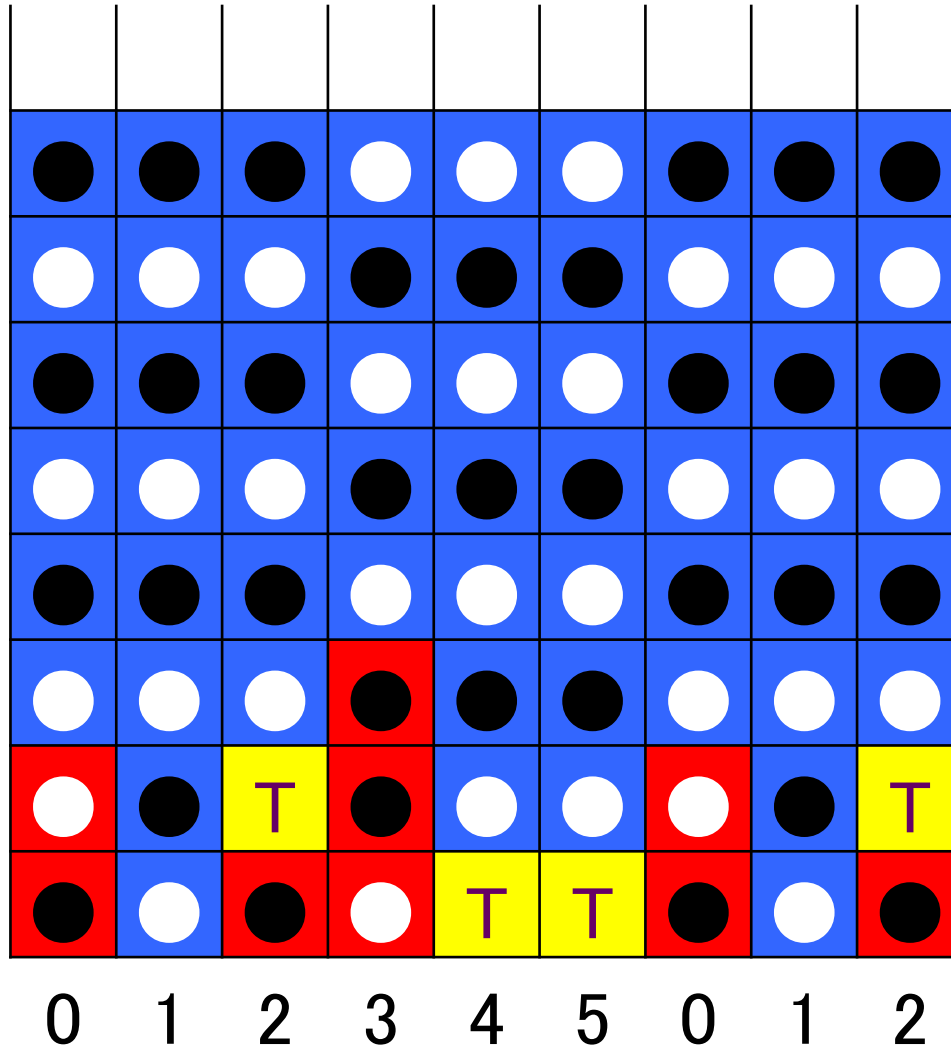
# Subtree Diagram 1



●0○0●2○3●3○1●0

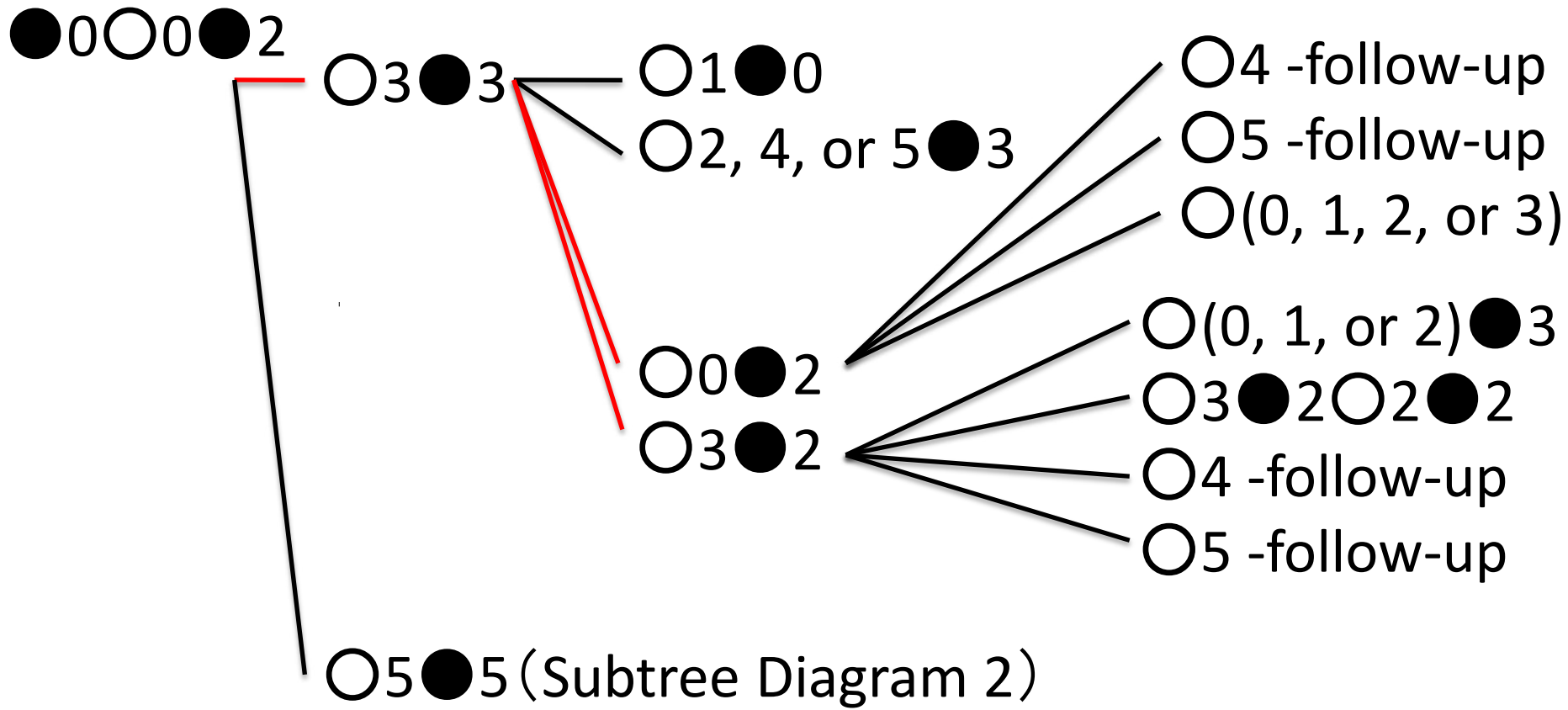


●0○0●2○3●3○(2, 4, or 5)●3

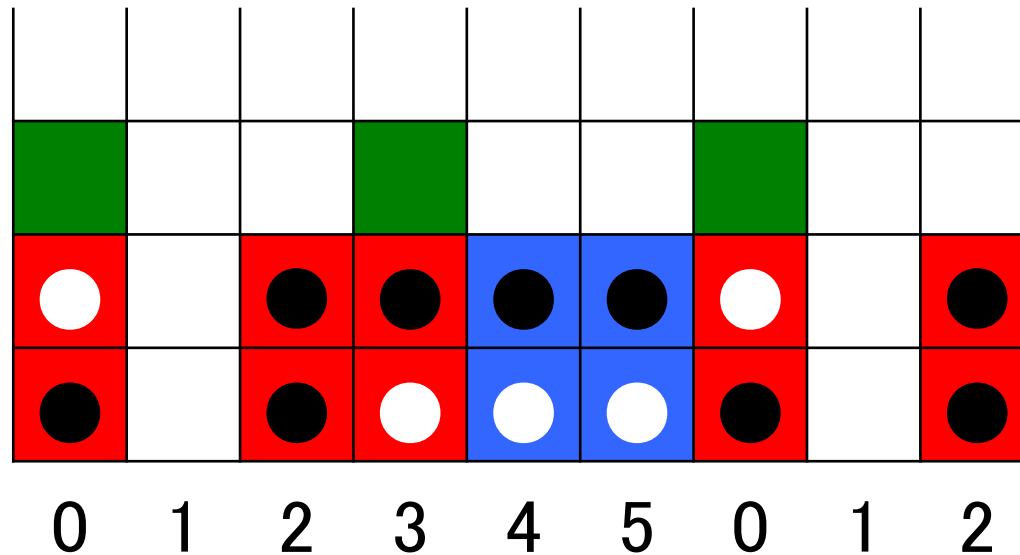




# Subtree Diagram 1

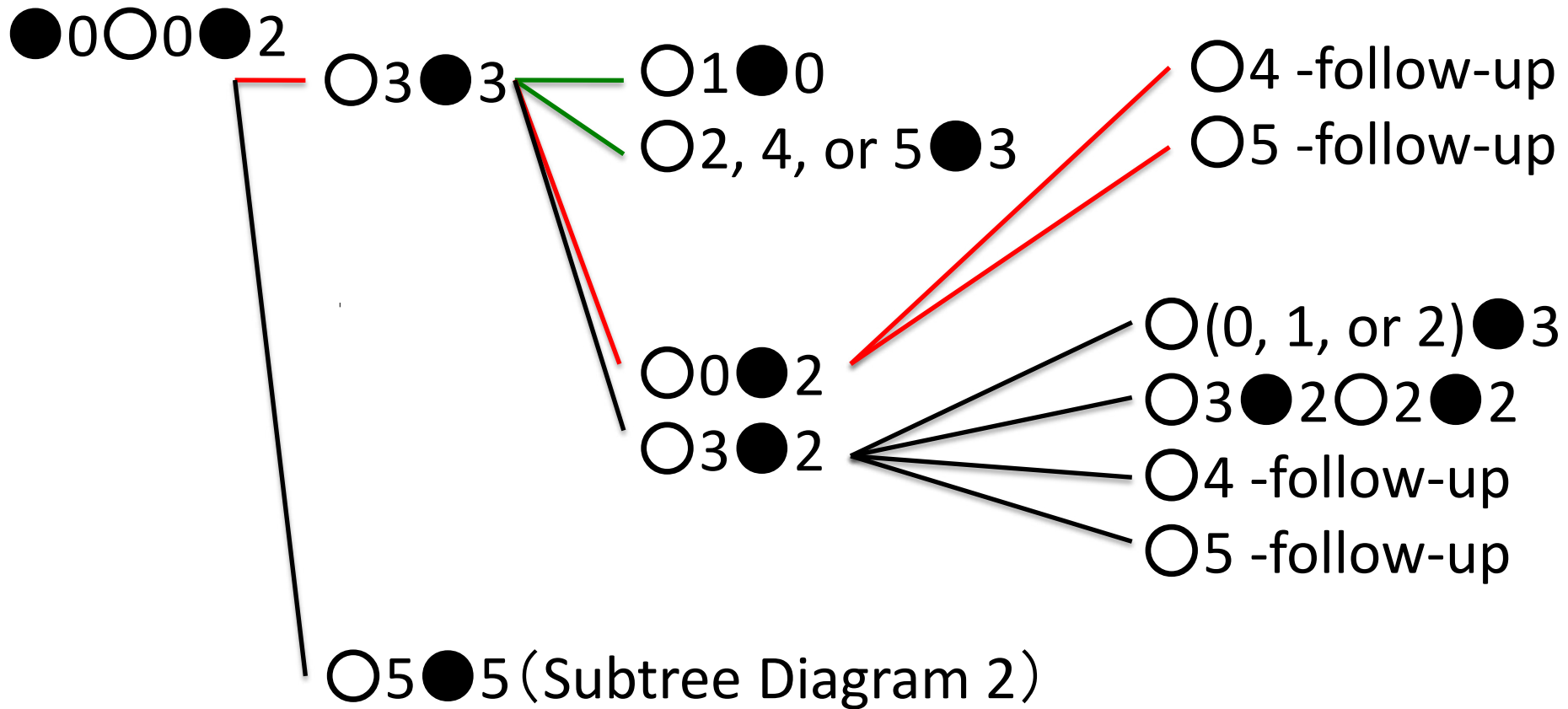


After ●0○0●2○3●3○0 or 3 (Green Cells),  
 Black Plays in Column 2 and Makes a Threat



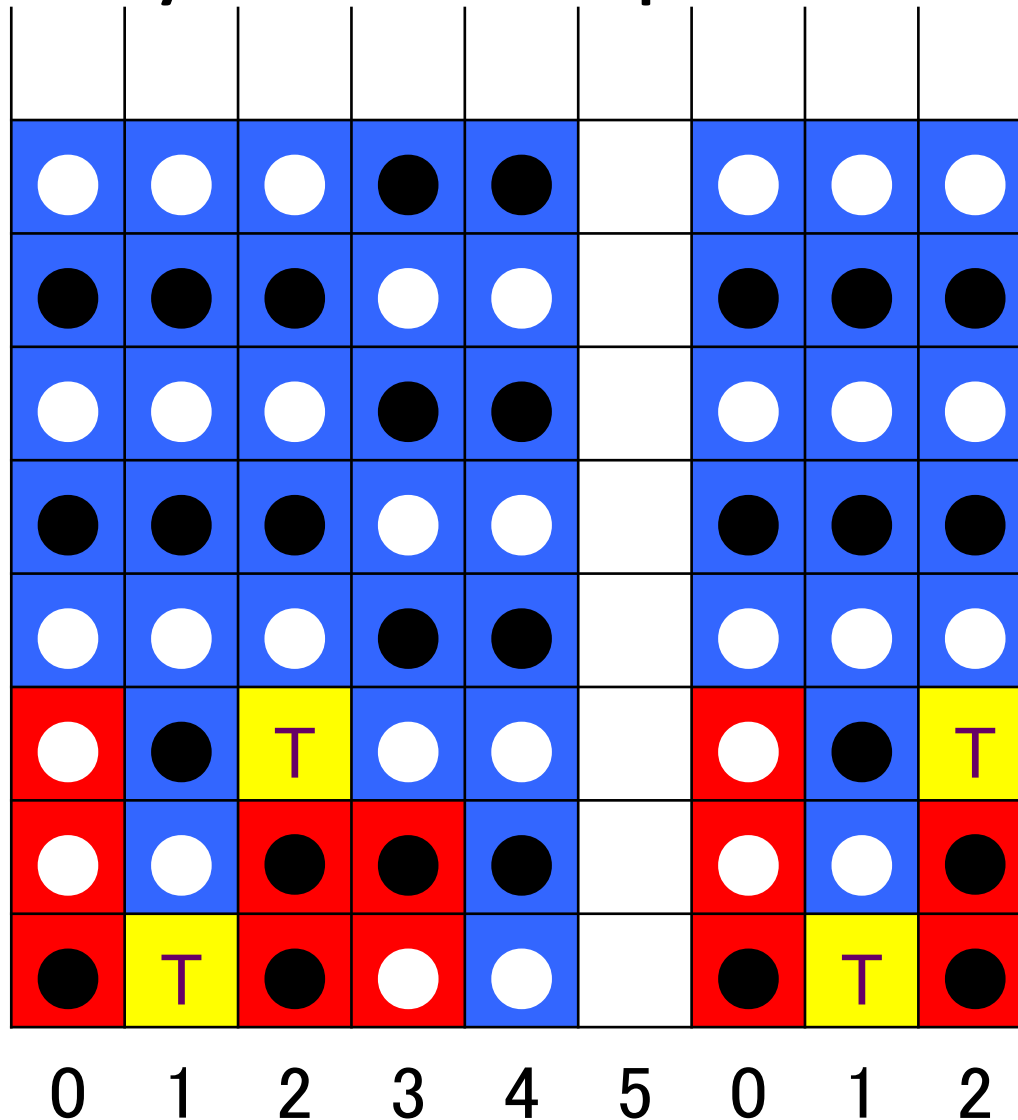
Black Plays Only Follow-up in Columns 4 and 5.

# Subtree Diagram 1

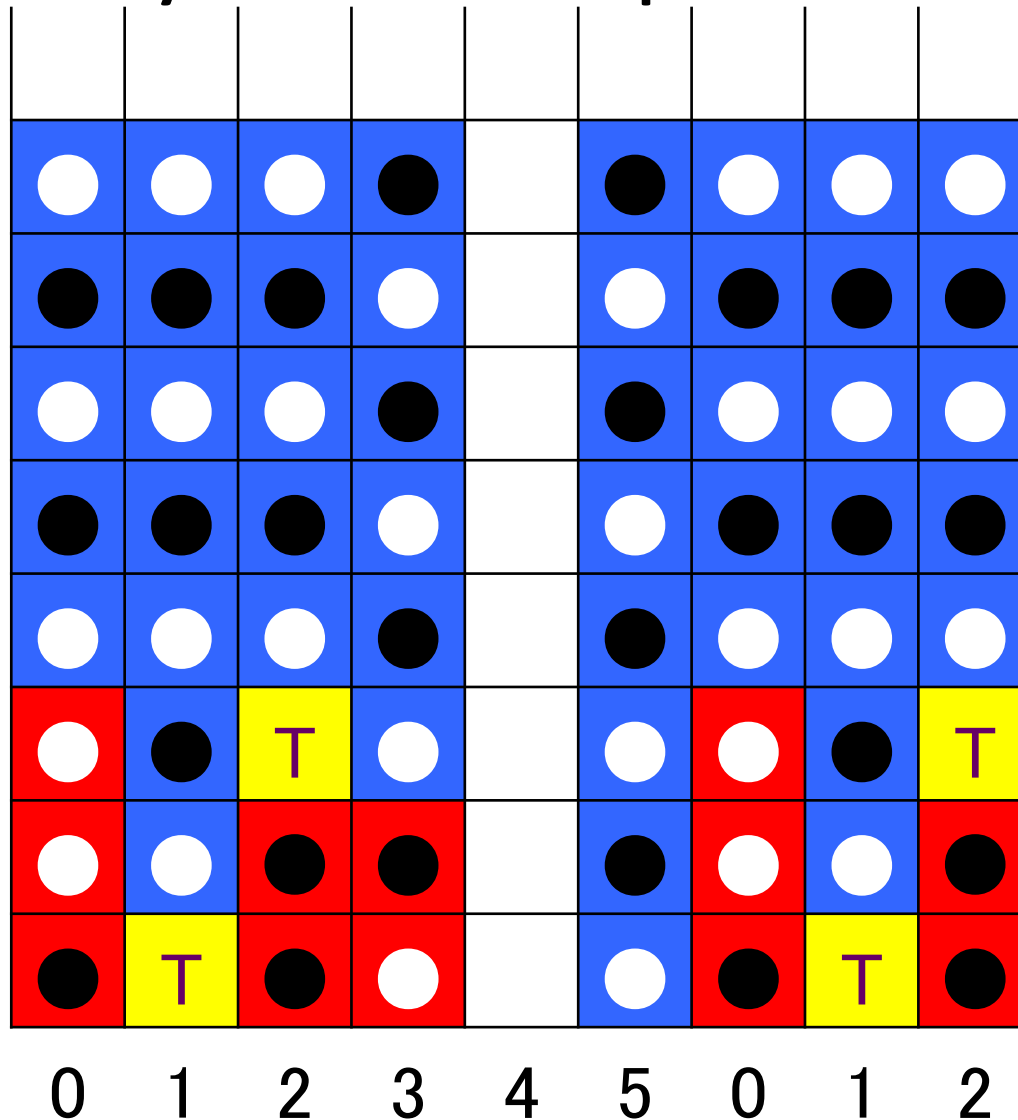


-follow-up: Black plays only follow-up.

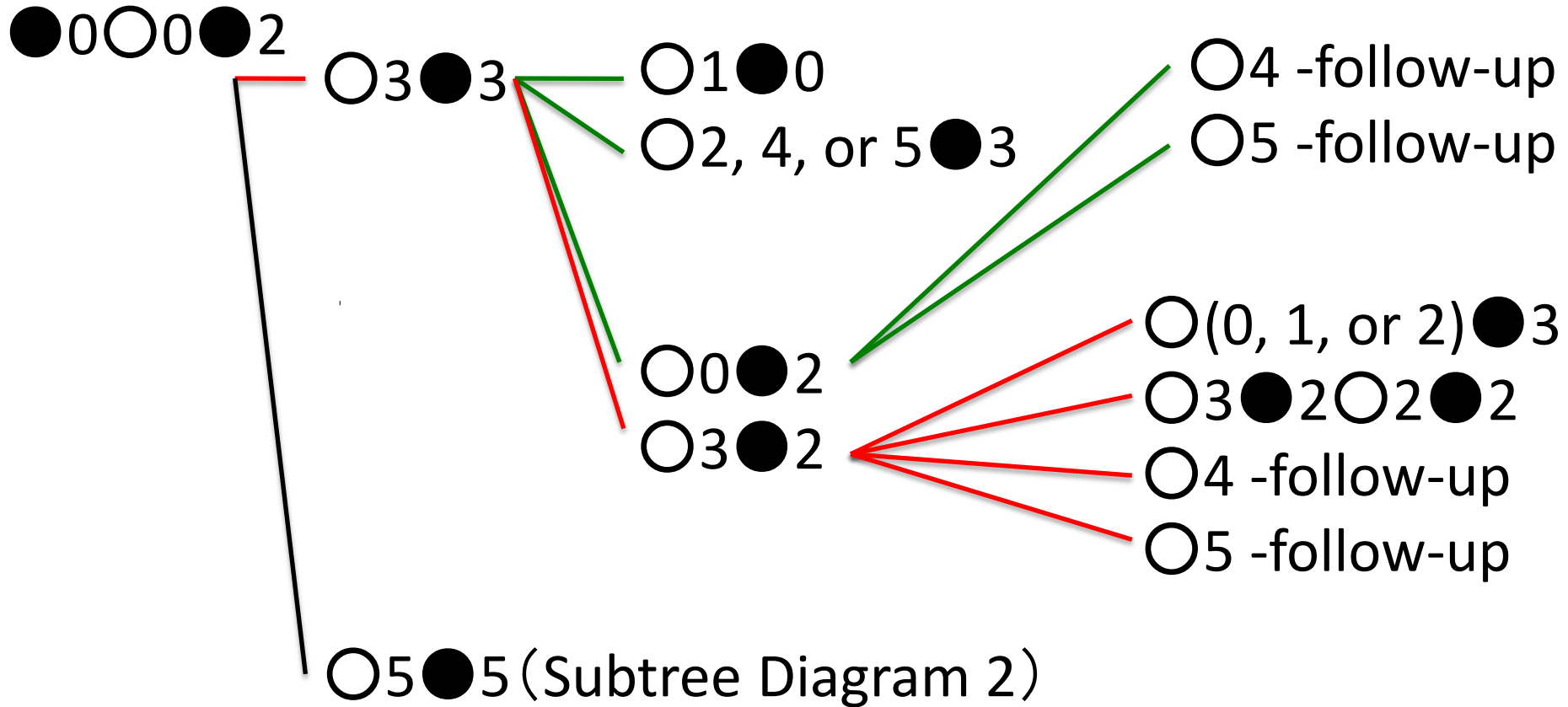
After ●0○0●2○3●3○0●2,  
 Black Plays Follow-up in Column 4



After ●0○0●2○3●3○0●2,  
 Black Plays Follow-up in Column 5

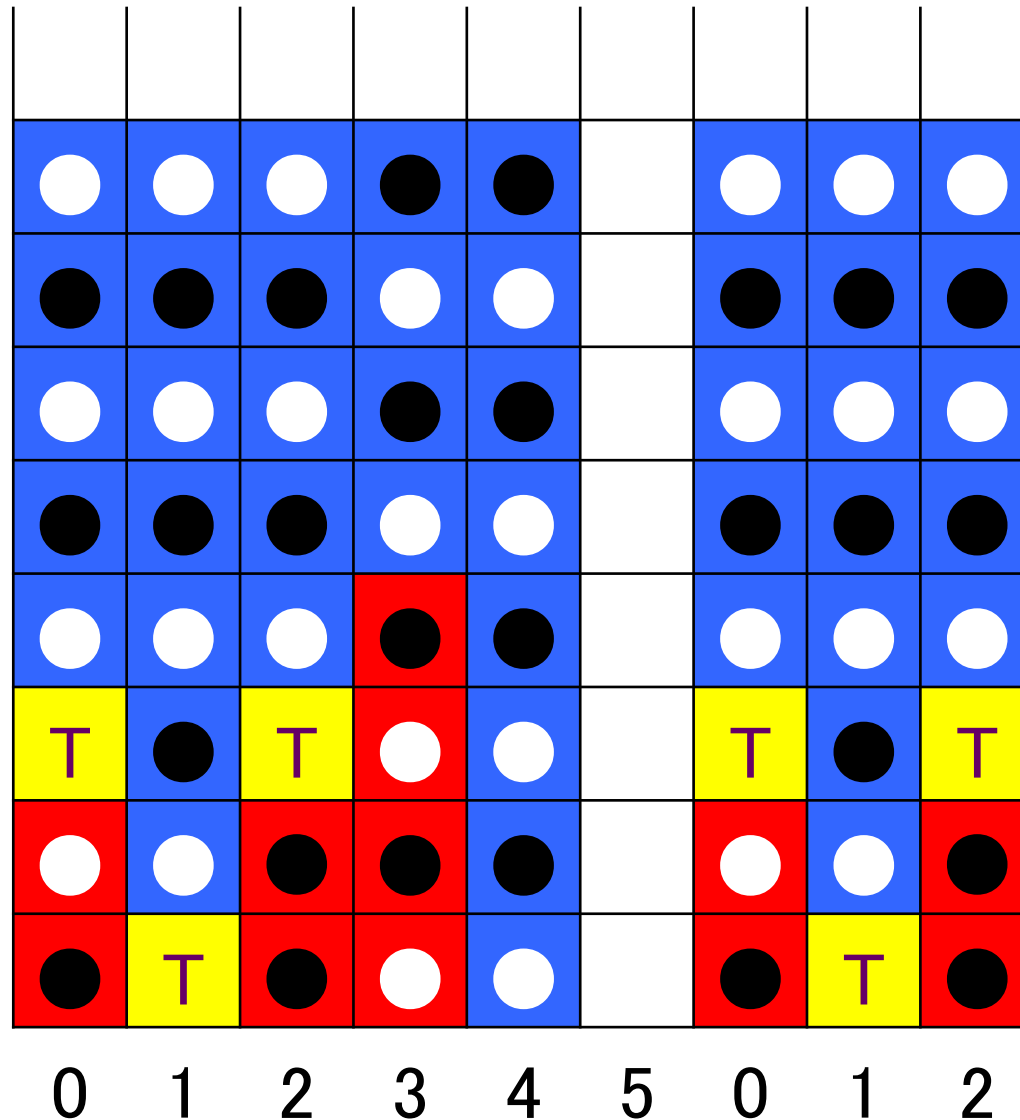


# Subtree Diagram 1

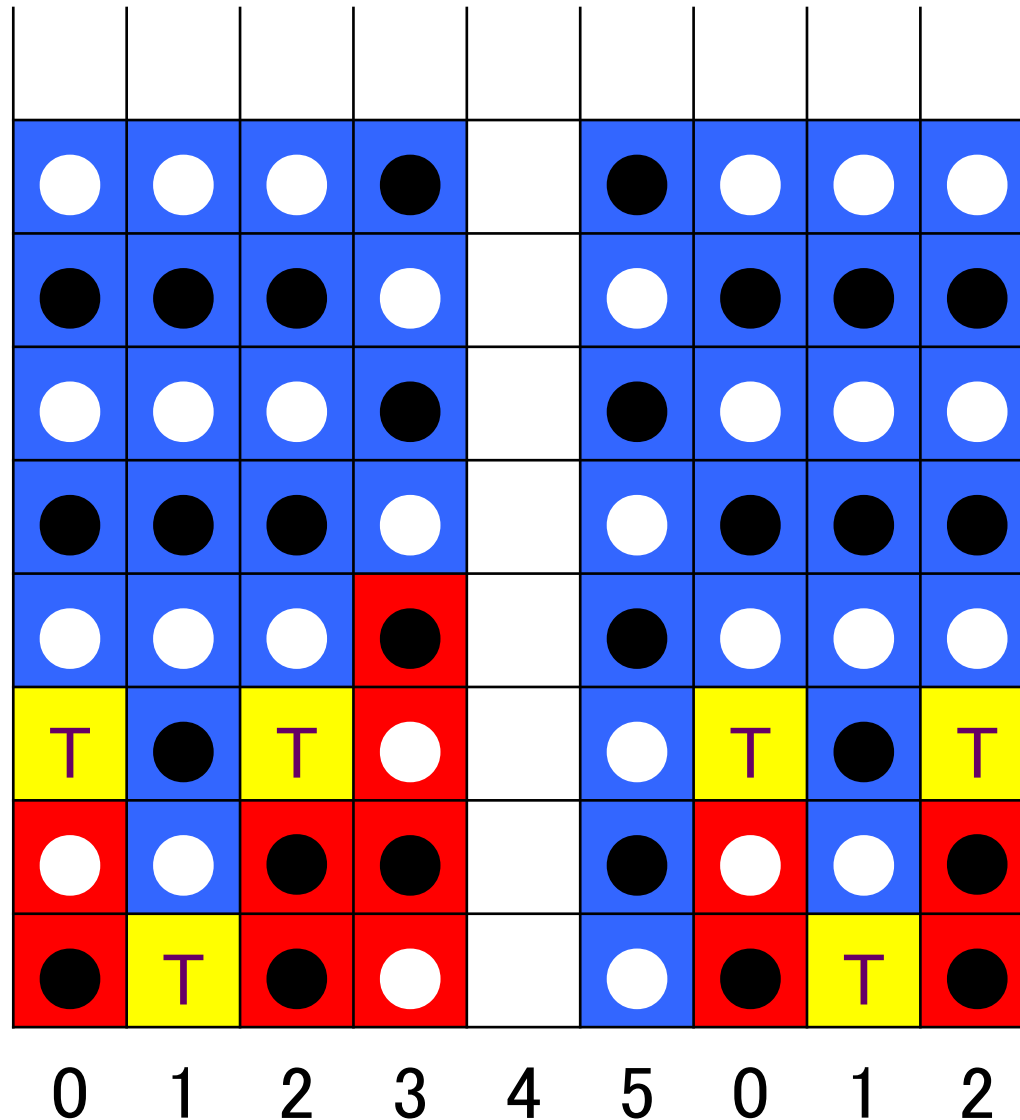


-follow-up: Black plays only follow-up.

After ●0○0●2○3●3○3●2, Black Plays  
 Follow-up in Column 4 and ○(0, 1, or 2)●3

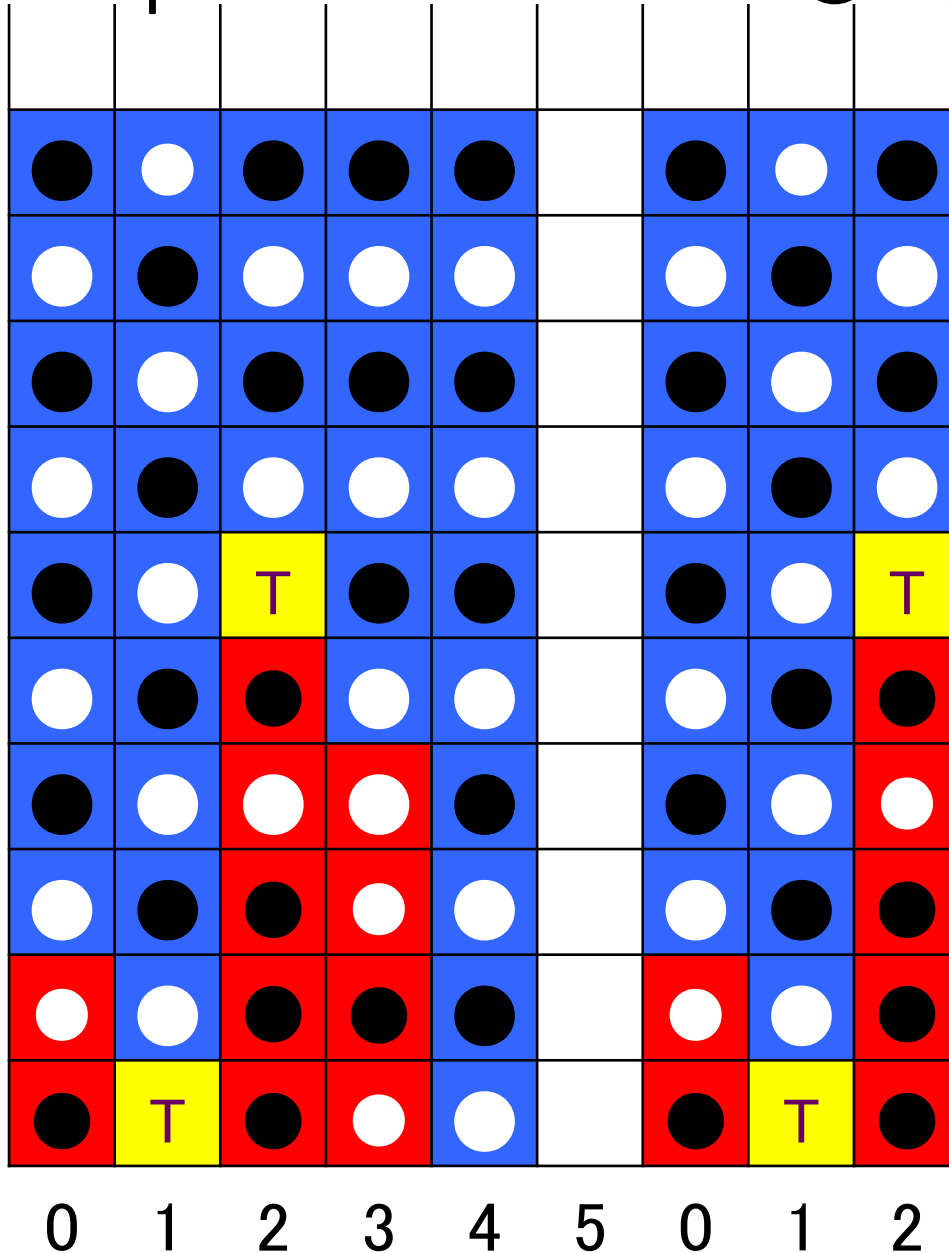


After ●0○0●2○3●3○3●2, Black Plays  
 Follow-up in Column 5 and ○(0, 1, or 2)●3

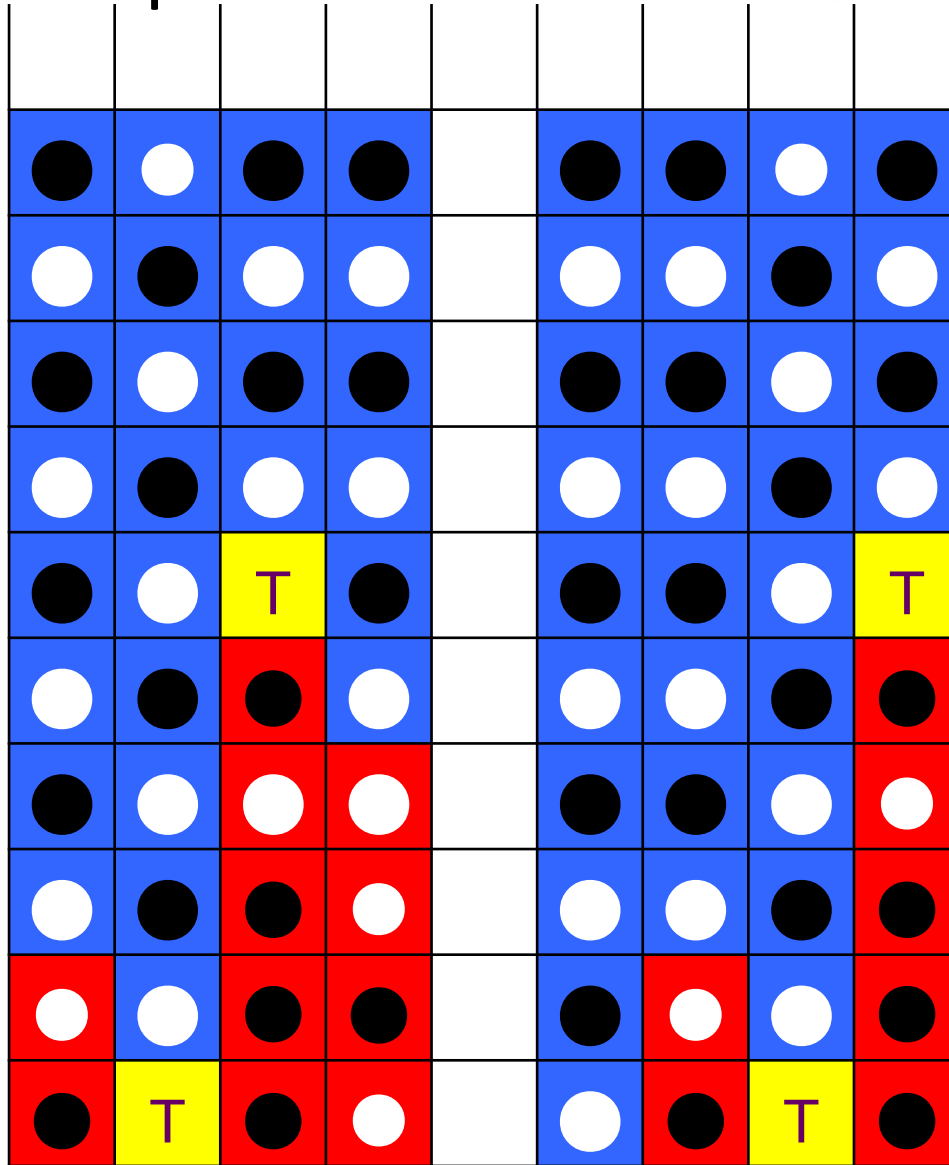




After ●0○0●2○3●3○3●2, Black  
 Plays Follow-up in Column 4 and ○3●2○2●2

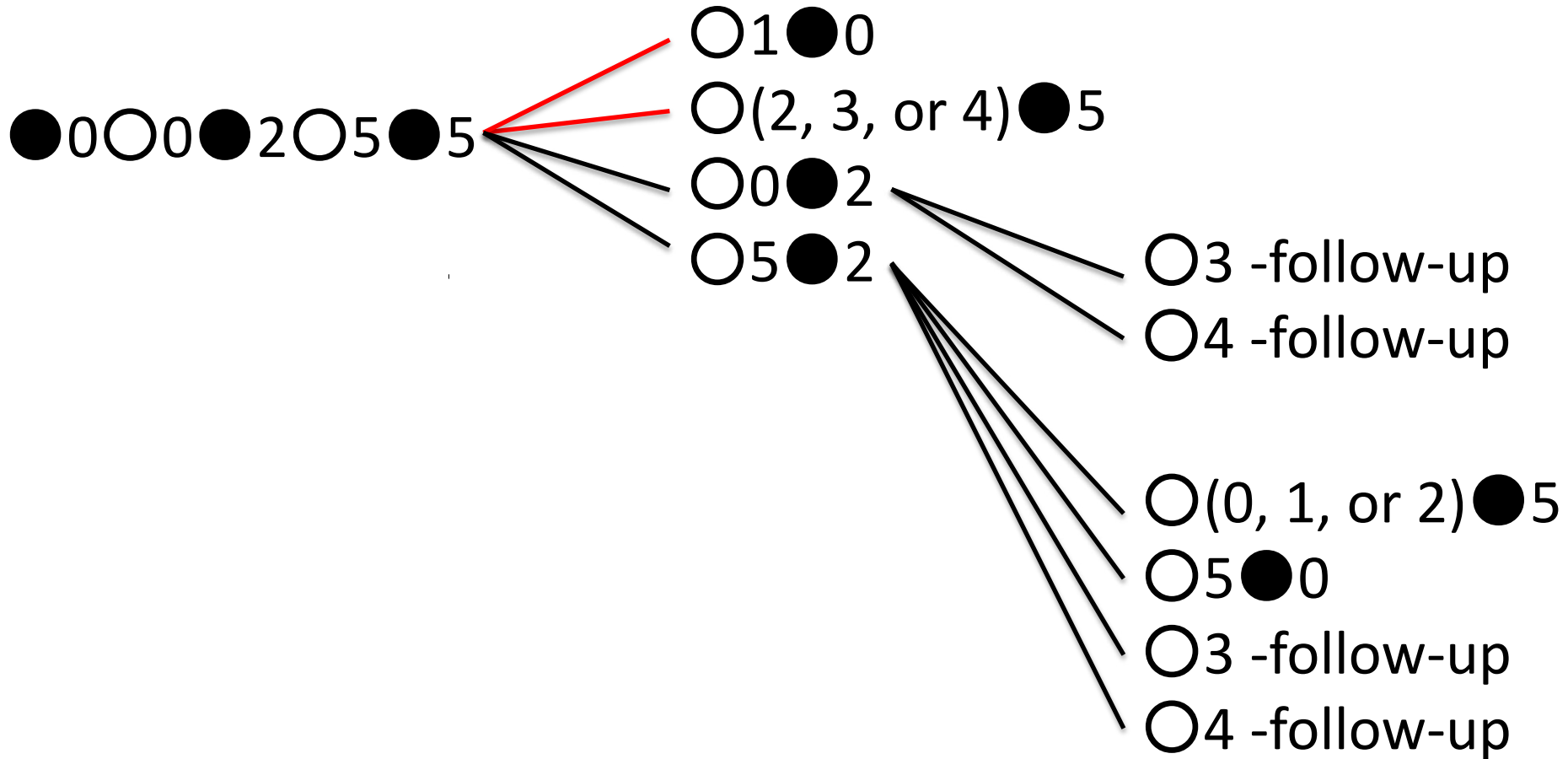


After ●0○0●2○3●3○3●2, Black  
 Plays Follow-up in Column 5 and ○3●2○2●2

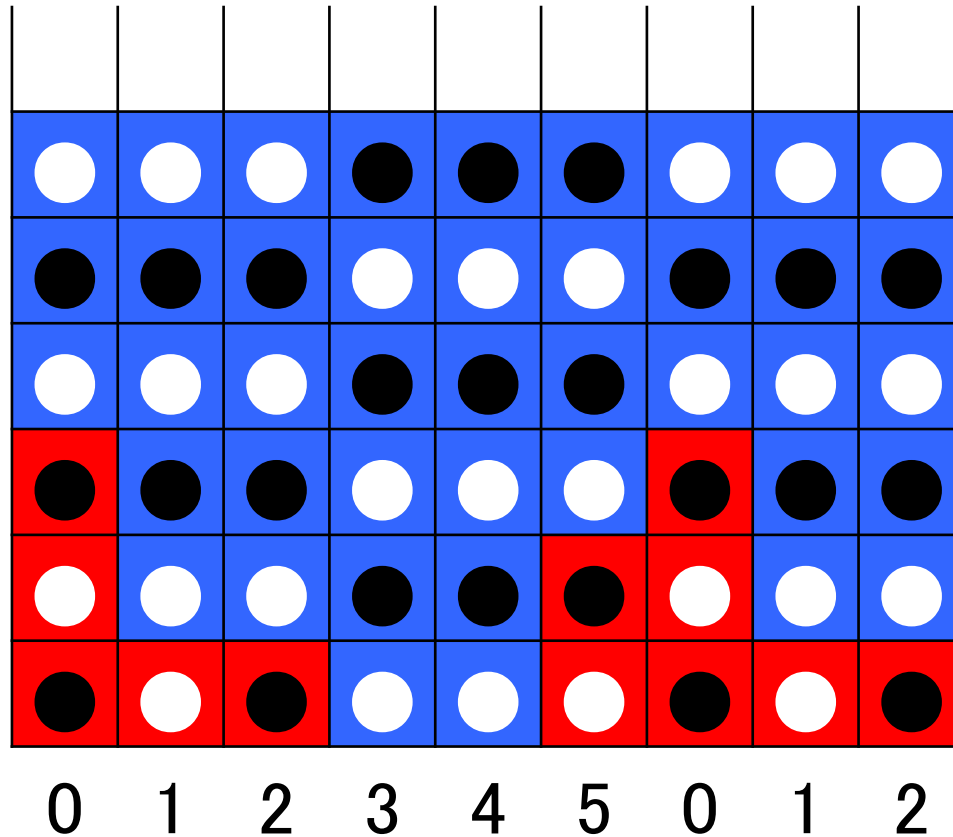


0 1 2 3 4 5 0 1 2

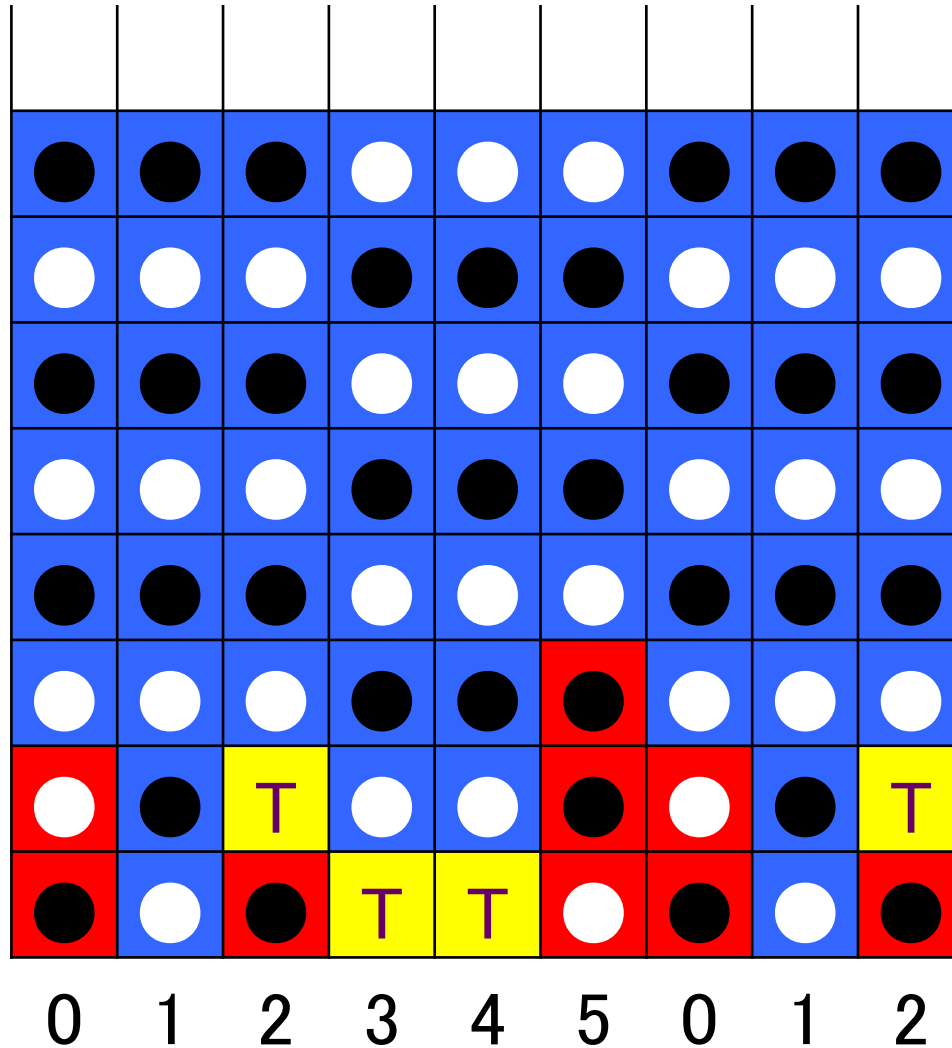
# Subtree Diagram 2



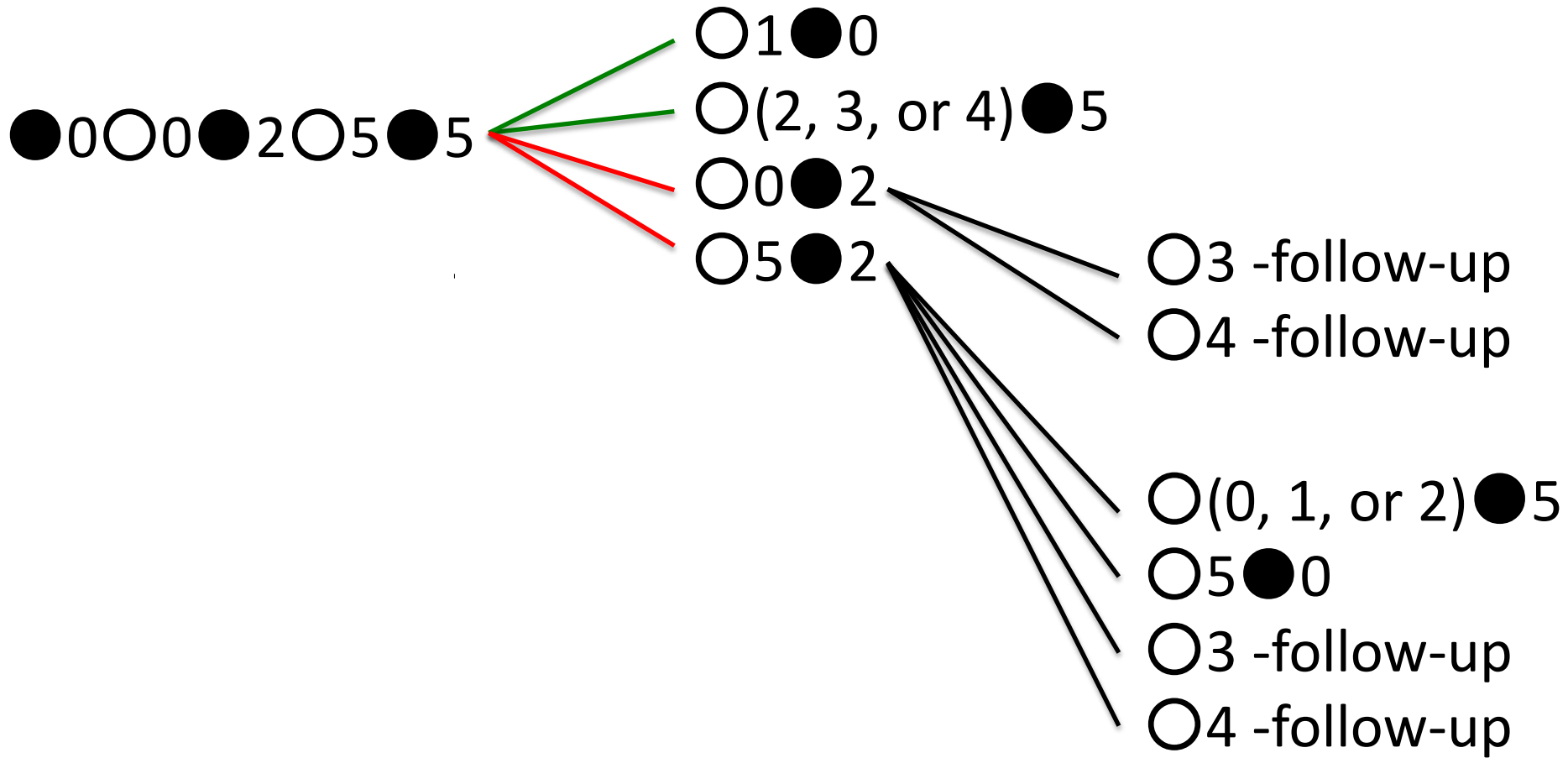
●0○0●2○5●5○1●0



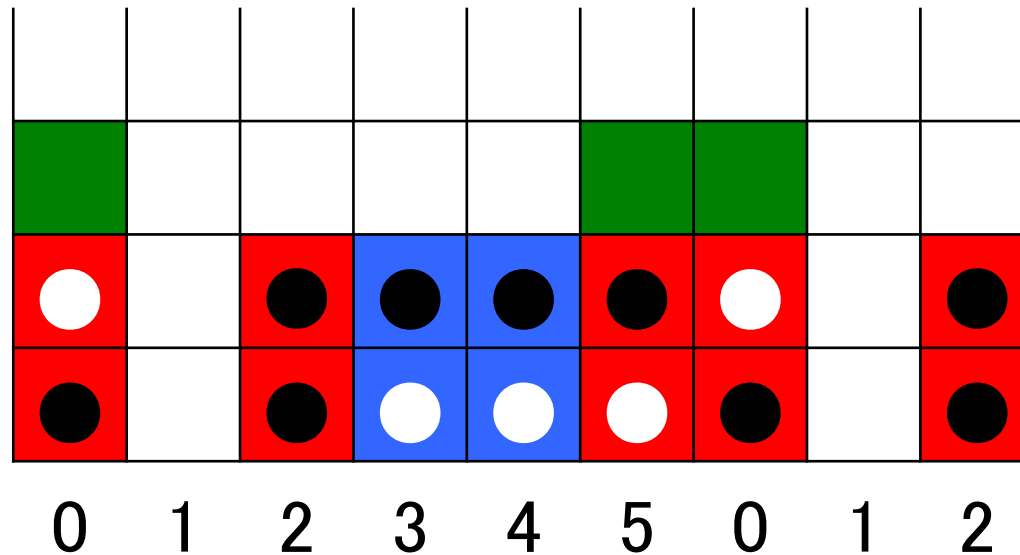
●0○0●2○5●5○(2, 3, or 4)●5



# Subtree Diagram 2

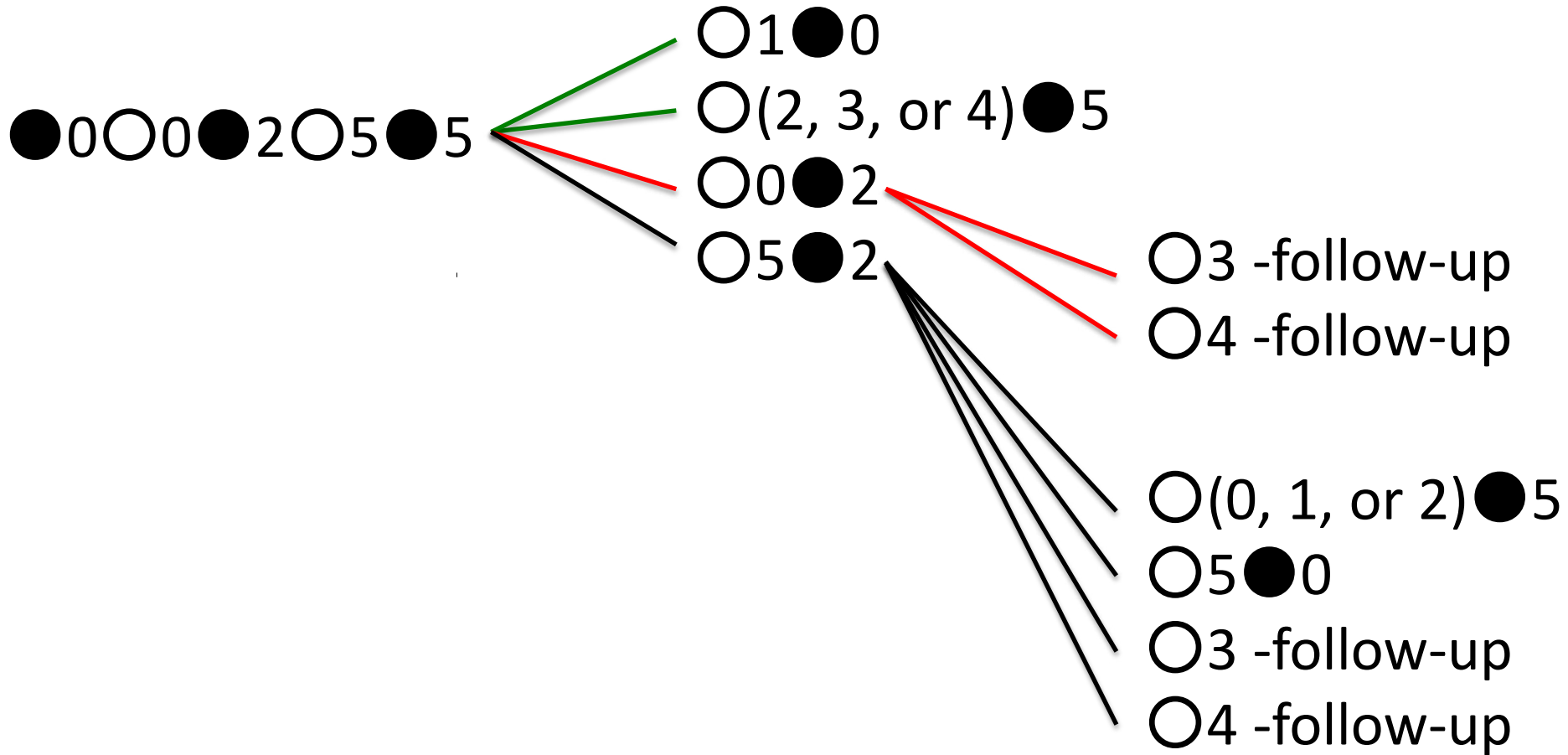


After ●0○0●2○5●5○0 or 5 (Green Cells),  
 Black Plays in Column 2 and Makes a Threat



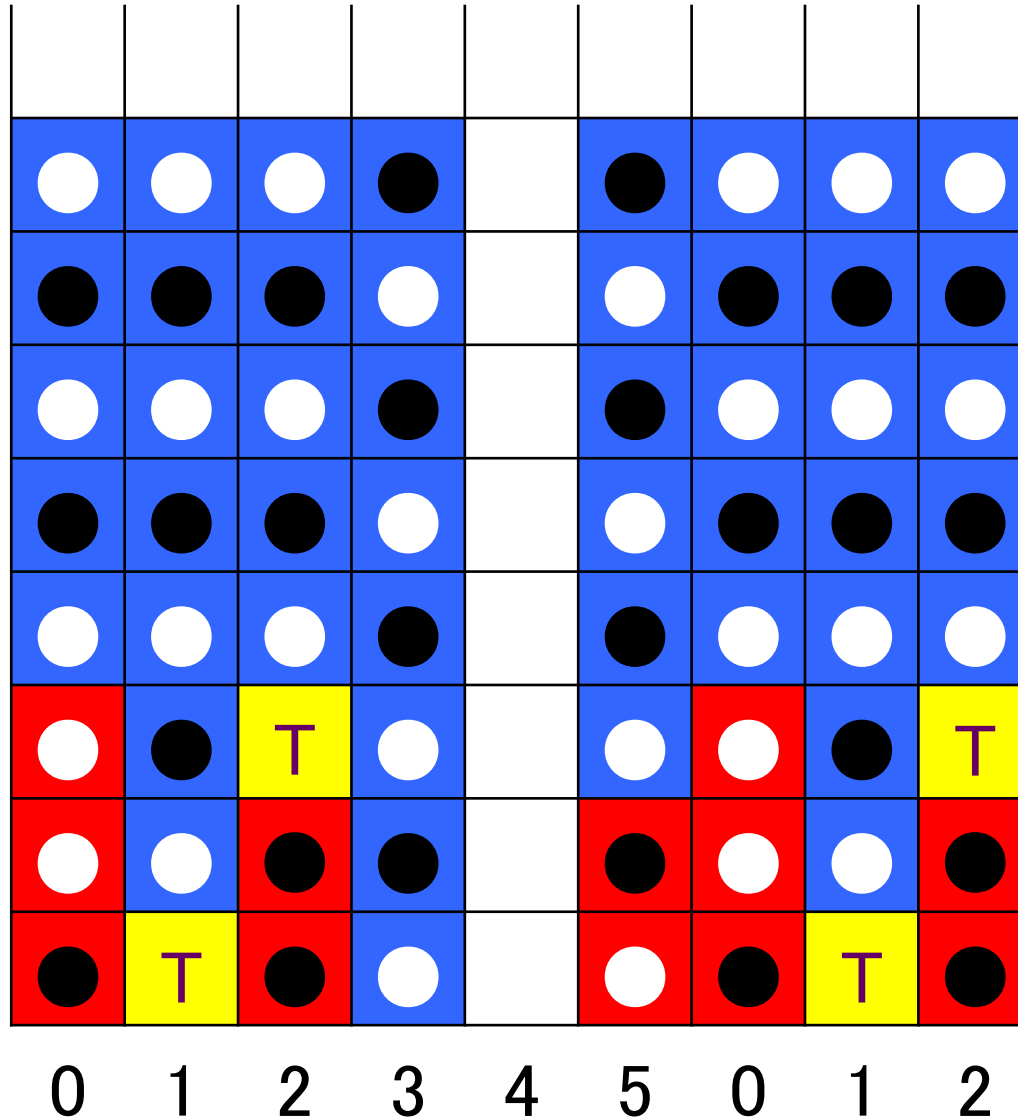
Black Plays Only Follow-up in Columns 3 and 4.

# Subtree Diagram 2

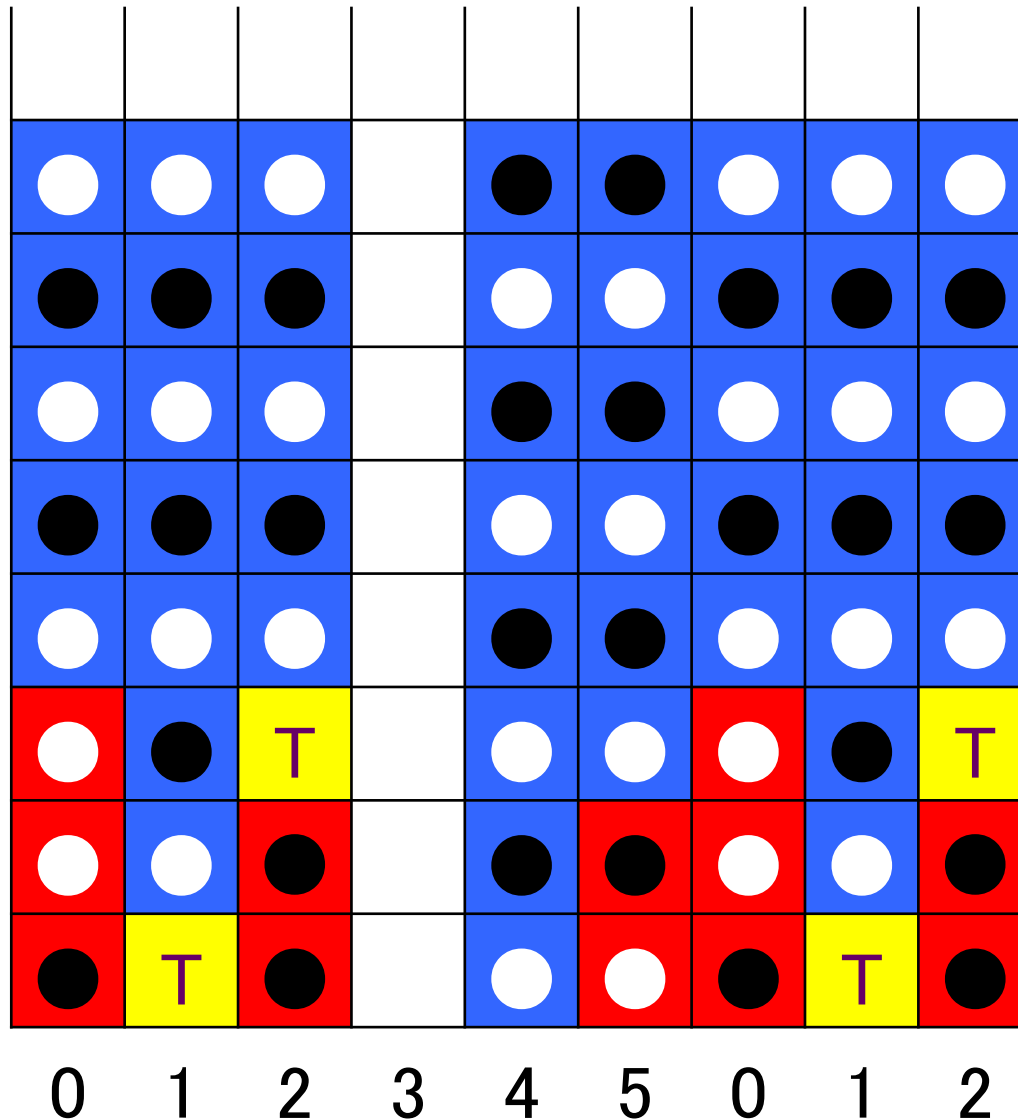




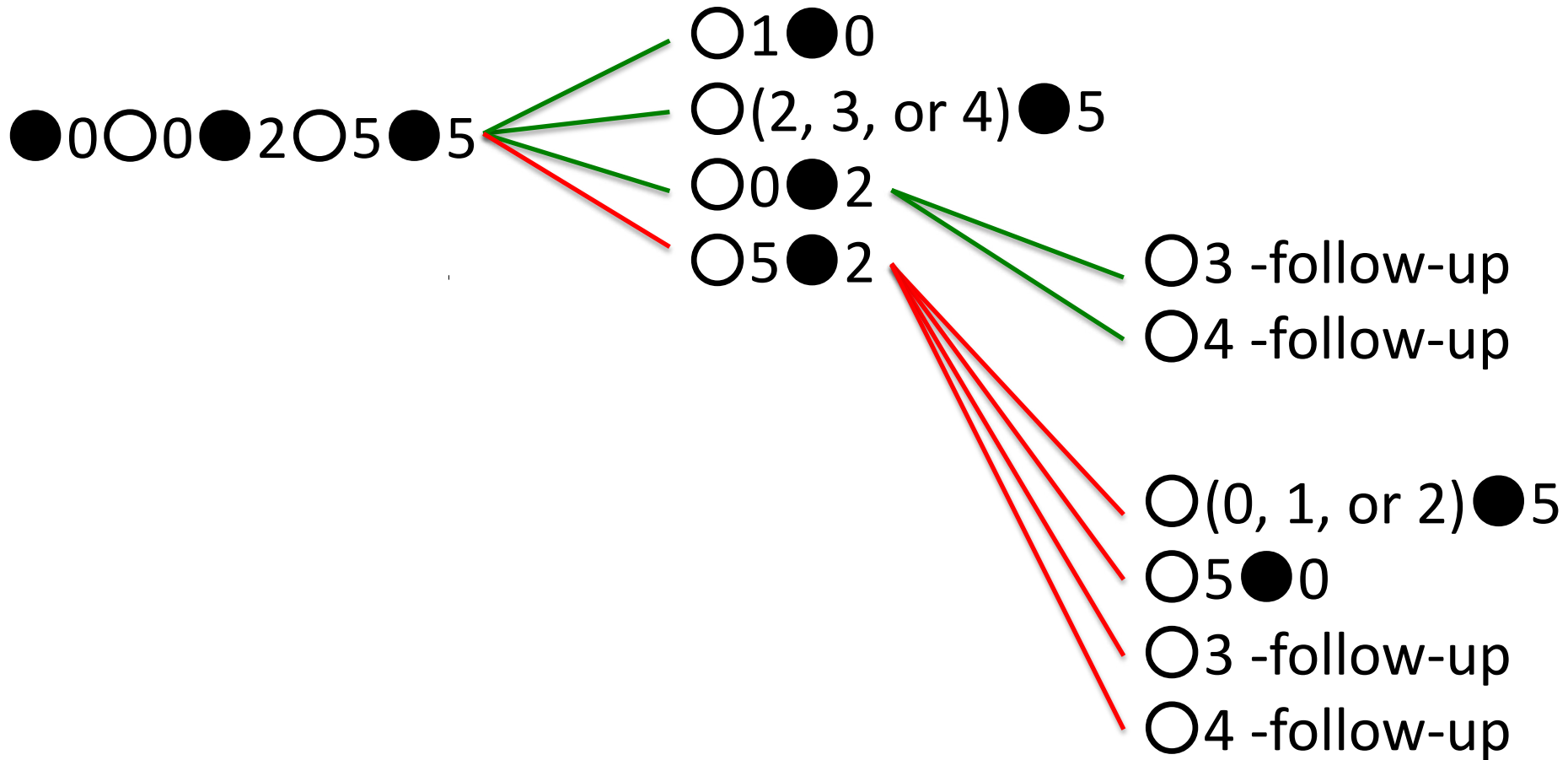
After ●0○0●2○5●5○0●2,  
 Black Plays Follow-up in Column 3



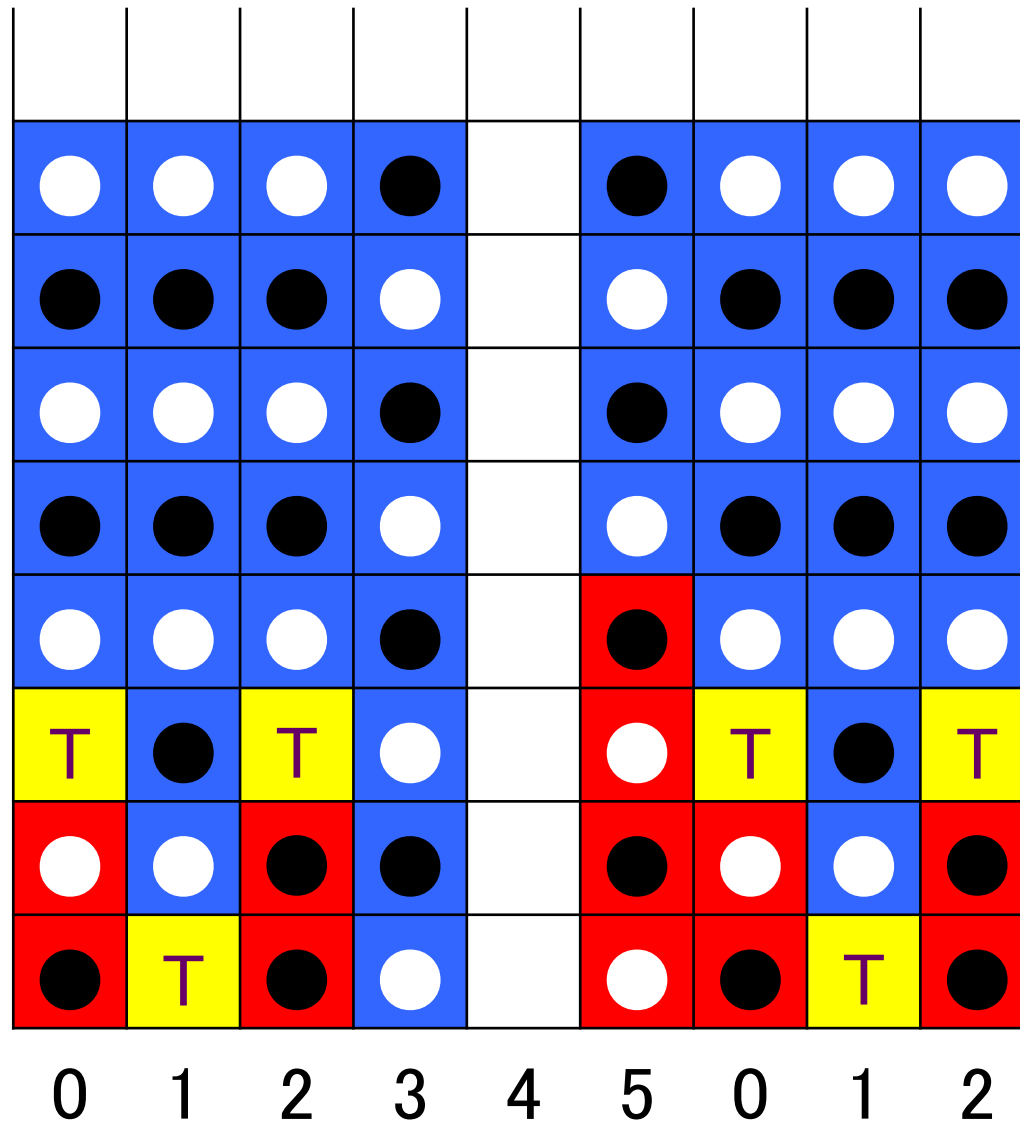
After ●0○0●2○5●5○0●2,  
 Black Plays Follow-up in Column 4



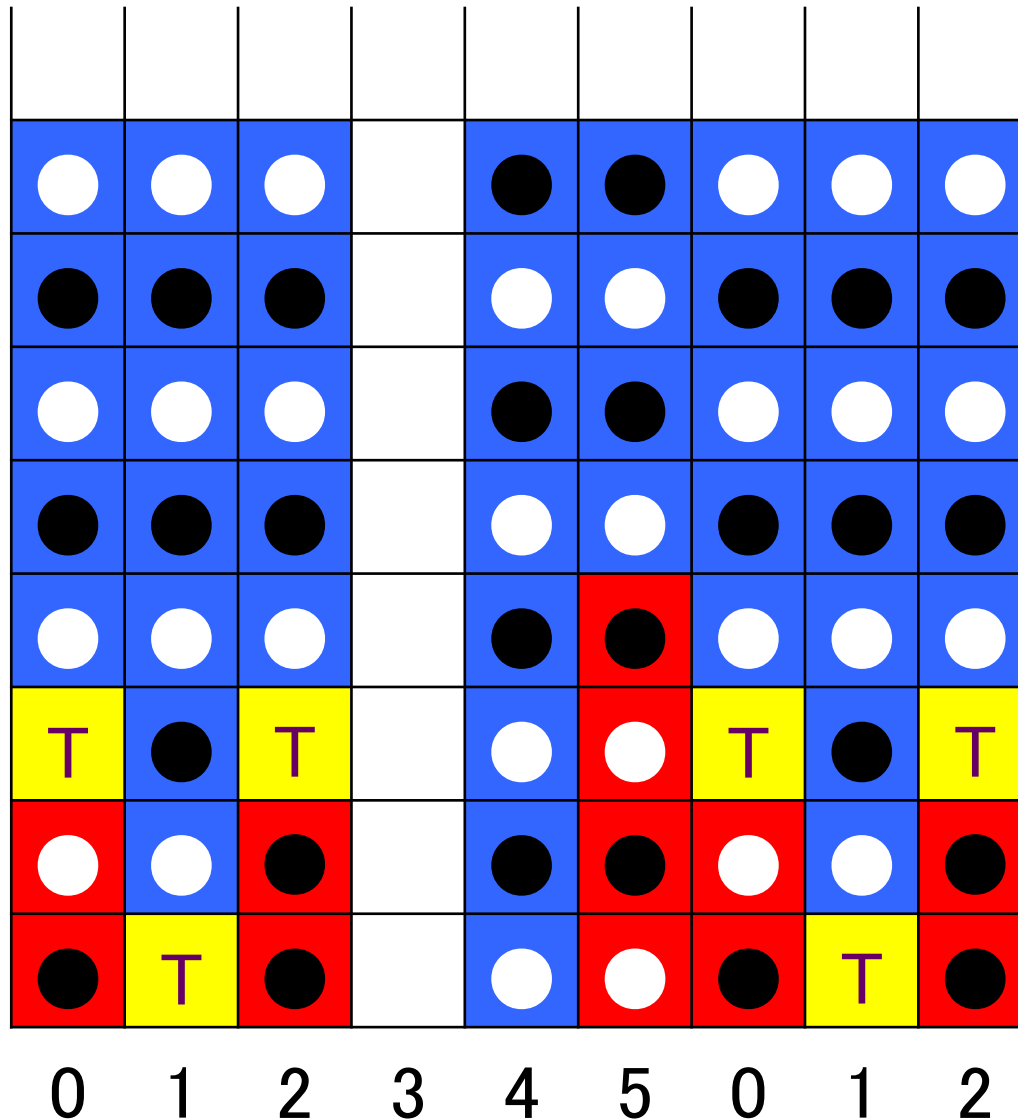
# Subtree Diagram 2



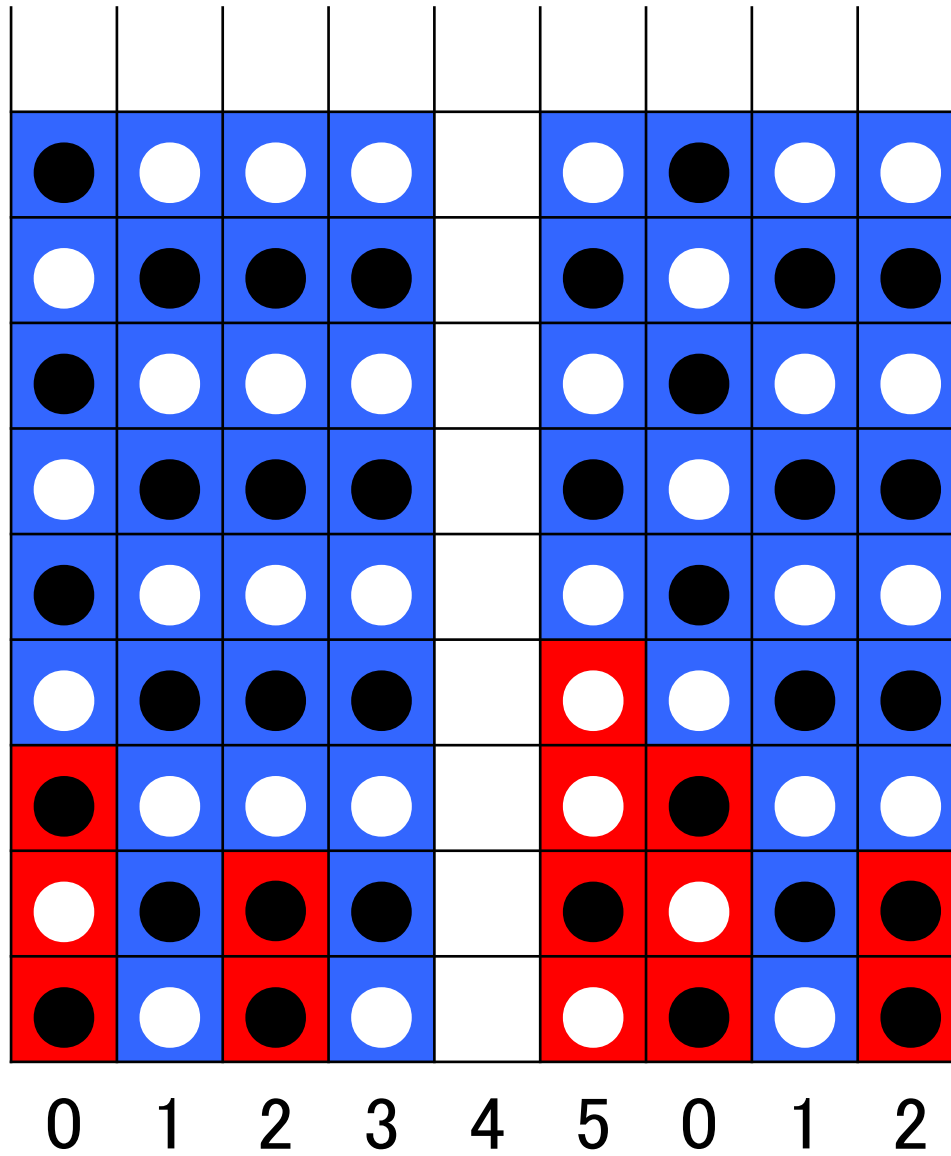
After ●0○0●25●5○5●2, Black Plays  
 Follow-up in Column 3 and ○(0, 1, or 2)●5



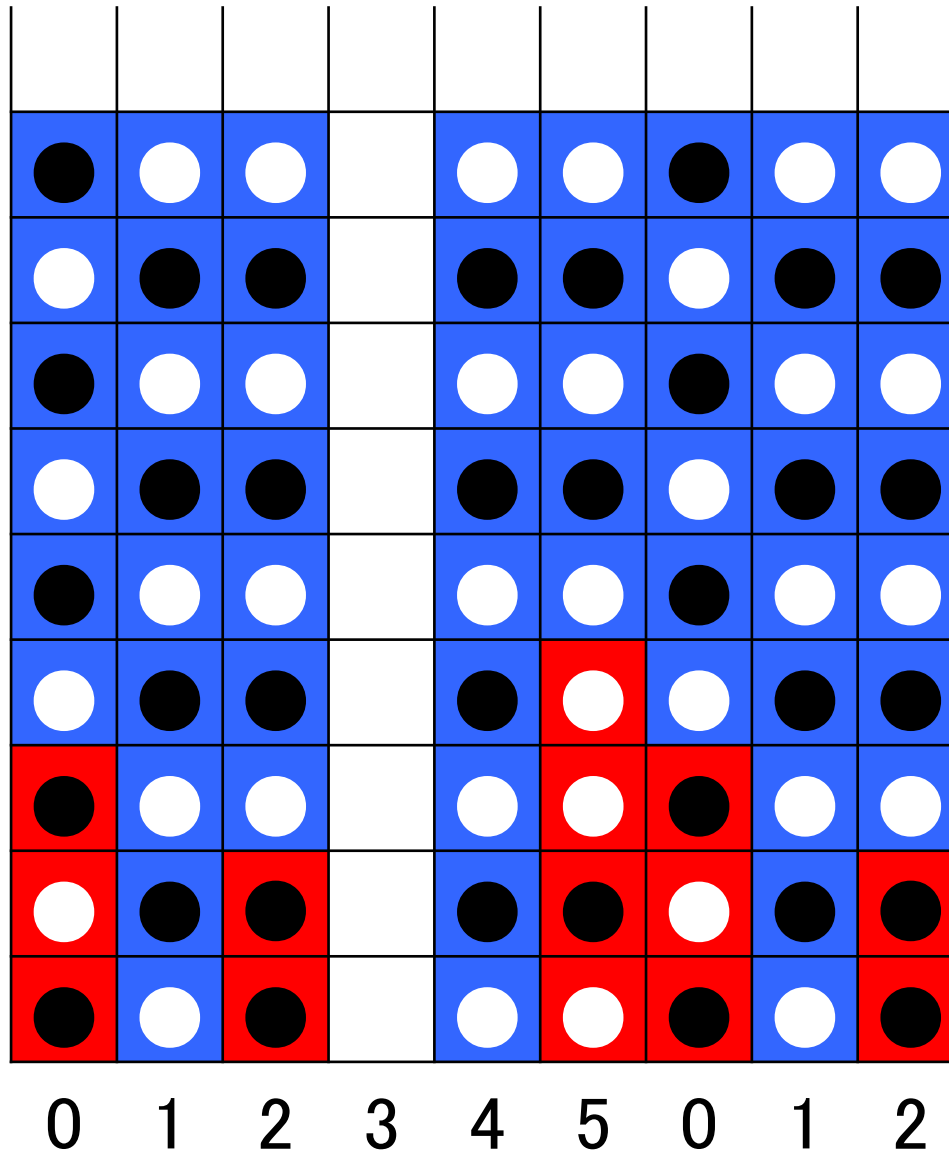
After ●0○0●2○5●5○5●2, Black Plays  
 Follow-up in Column 4 and ○(0, 1, or 2)●5



After ●0○0●2○5●5○5●2, Black  
Plays Follow-up in Column 3 and ○5●0



After ●0○0●2○5●5○5●2, Black  
Plays Follow-up in Column 4 and ○5●0



# Conclusion



# Threats

- Both players cannot achieve a Connect4 in cannot-lose strategies of previous work.
- Threats are useful for Black's cannot-lose strategies for widths 2 and 6.
- Black's cannot-lose strategies for all widths are discovered.
- Threats will be useful for discovering White's cannot-lose strategies for widths 6 and 11.