# Rook Jumping Mazes:

## a Computer Science design project

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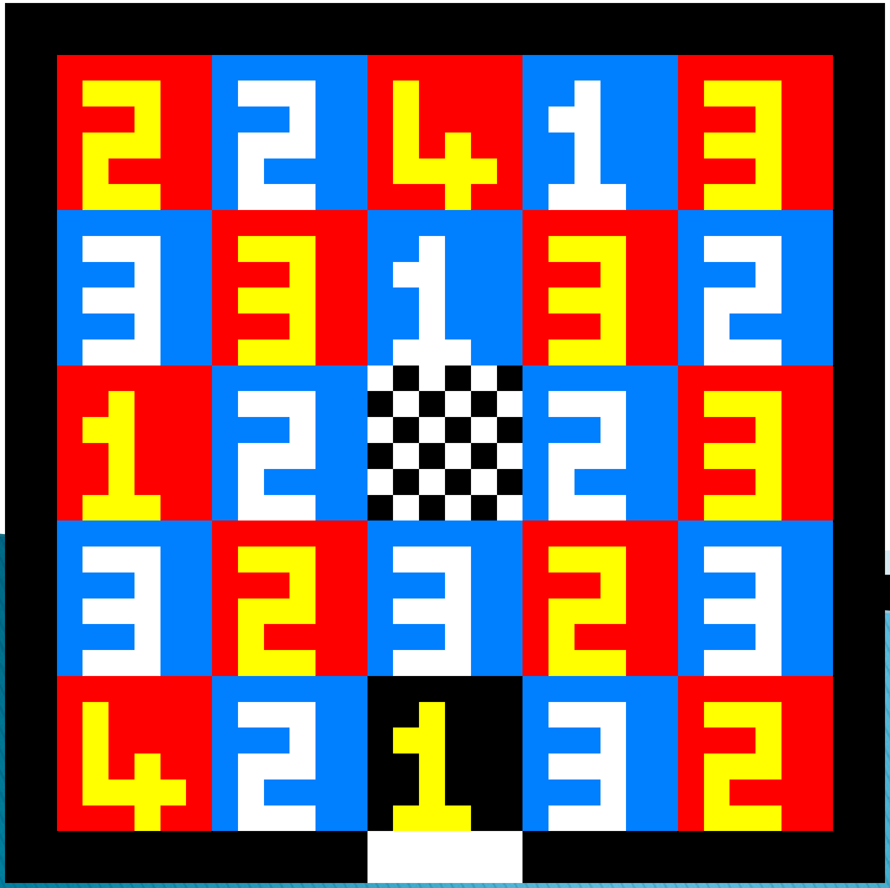
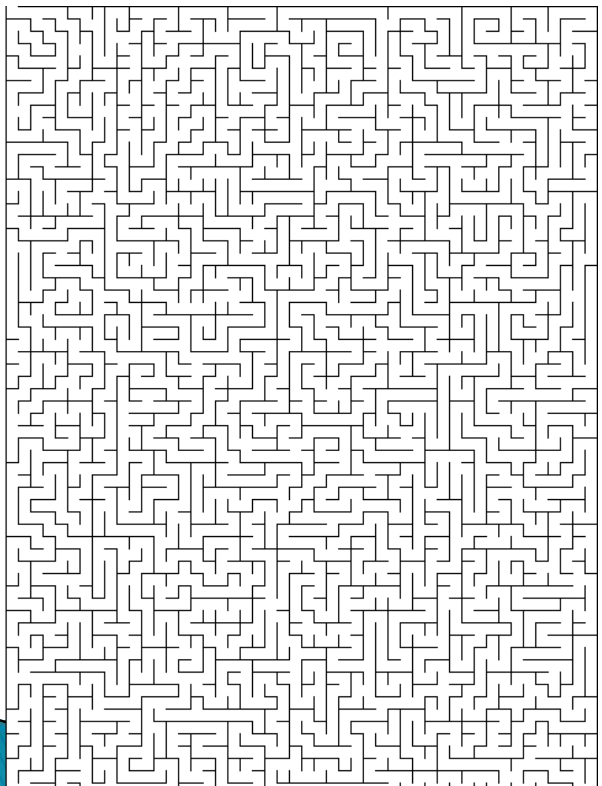



Maze © Adrian Fisher

# Traditional Mazes

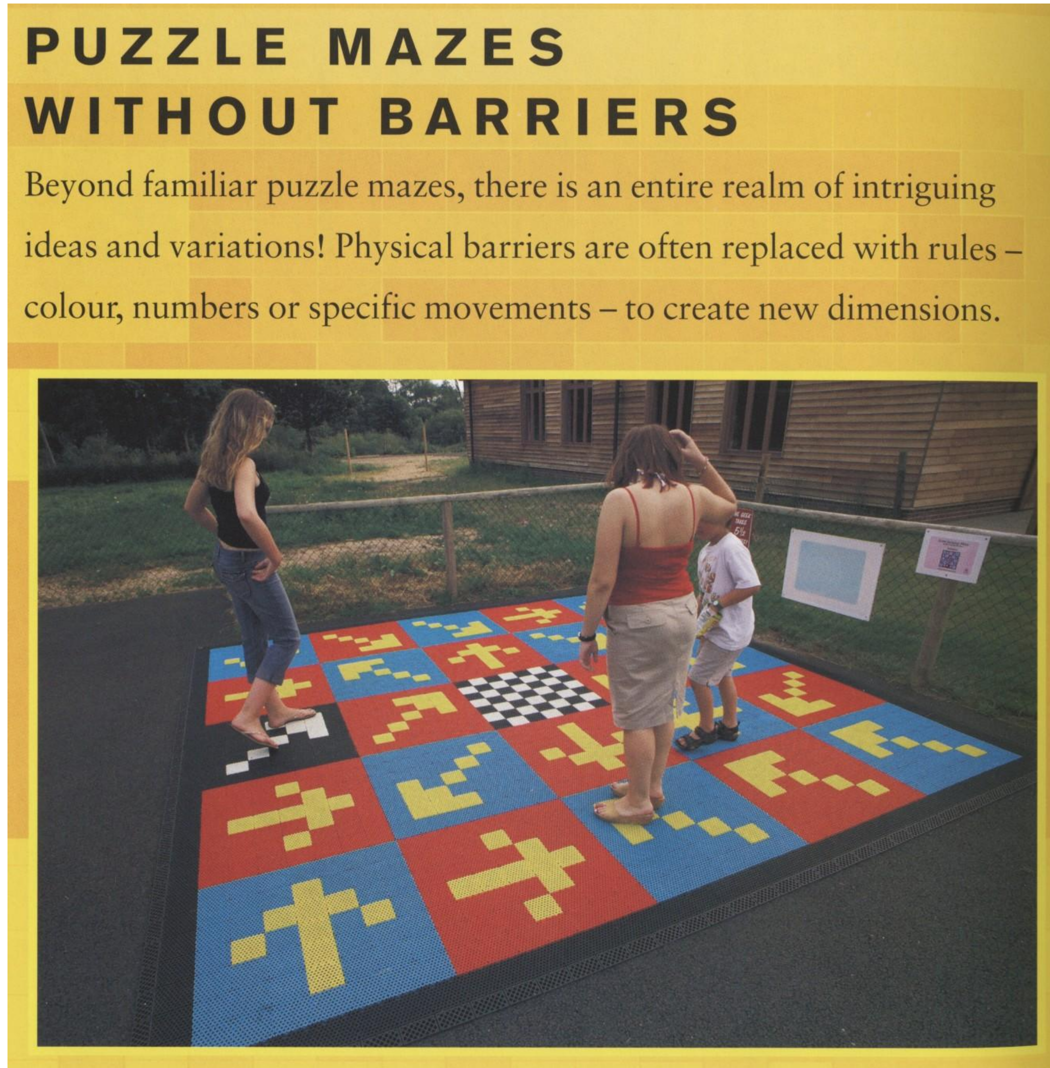
- Lines, hedges, corn rows, etc. delineate paths
- Find a path from start to goal



From "The Amazing Book of Mazes" by Adrian Fisher

# Logic Mazes, Quick Mazes

- Smaller, more complex states and/or transitions
- Find a “path” (state sequence) from start state to goal state
- Small footprint, novel rules, fast accessibility and solving

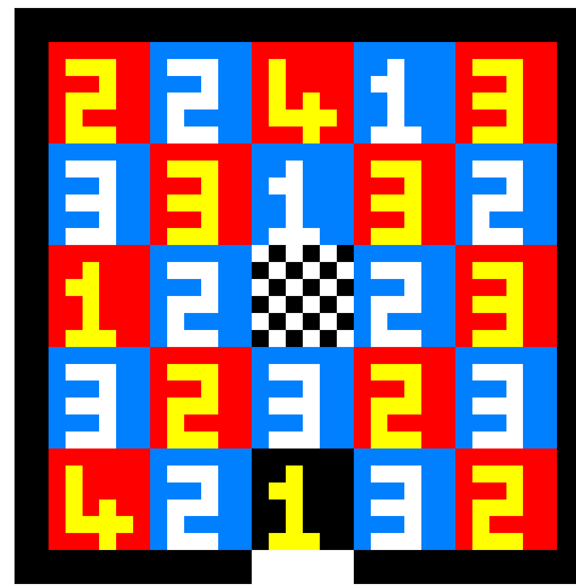

PUZZLE MAZES WITHOUT BARRIERS

Beyond familiar puzzle mazes, there is an entire realm of intriguing ideas and variations! Physical barriers are often replaced with rules – colour, numbers or specific movements – to create new dimensions.

From “The Amazing Book of Mazes” by Adrian Fisher

# Rook Jumping Maze

- Basic form: square grid, start state (square), goal state, jump numbers for each non-goal state.
- Jump number: Move *exactly* that many squares forward, backward, left, right. (*Not* diagonally.)
- Objectives:
  - Find a path from start to goal
  - Find the shortest of these paths



# Sample Mazes

| | | | | | |
|---|---|---|---|---|---|
| 3 0 START | 5 | 3 | 4 | 3 | 2 |
| 5 | 2 | 2 | 4 | 2 | 2 |
| 4 | 1 | 3 | 2 | 3 | 2 |
| 3 | 5 | 1 | 3 | 2 | 3 |
| 1 | 2 | 2 | 2 | 4 | 2 |
| 4 | 4 | 5 | 5 | 4 | 0 FINISH |

RJ 6001

Adrian Fisher "Rook Jumping Maze"

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 START | 2 | 7 | 4 | 2 | 4 | 4 | 2 |
| 2 | 2 | 2 | 5 | 4 | 2 | 1 | 6 |
| 3 | 4 | 2 | 4 | 2 | 5 | 5 | 3 |
| 4 | 5 | 4 | 3 | 2 | 2 | 3 | 4 |
| 2 | 1 | 2 | 3 | 4 | 4 | 2 | 3 |
| 2 | 4 | 3 | 2 | 3 | 3 | 6 | 4 |
| 6 | 4 | 2 | 3 | 3 | 2 | 6 | 3 |
| 6 | 7 | 4 | 3 | 3 | 2 | 2 | GOAL |

Robert Abbott "No U-Turn Number Maze" Forward, left, right but *not* back

# Puzzle Design as Search

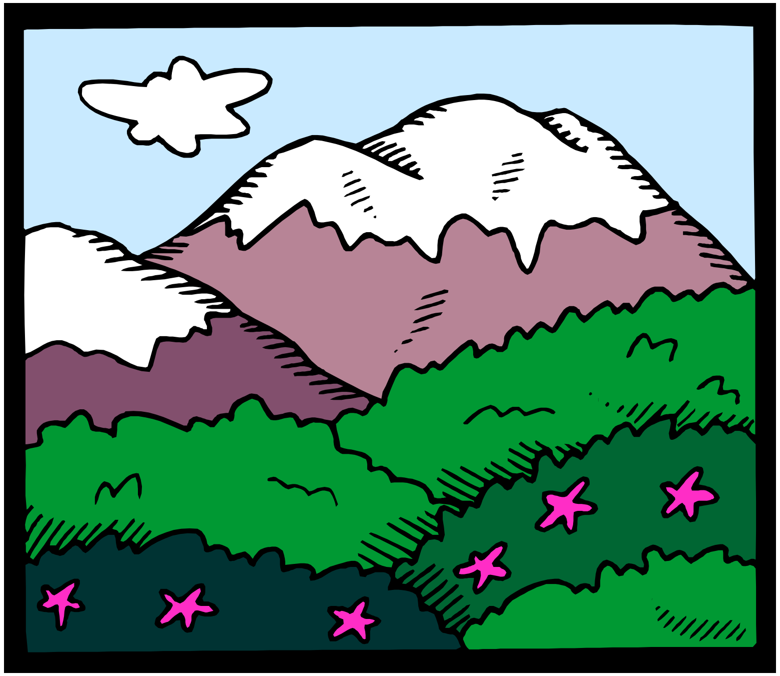
- The number of possible 5 × 5 rook jumping mazes configurations with a center goal: $4^{16} * 3^8 > 2.8 \times 10^{13}$ (a lot)
- The number of possible $n \times n$ mazes is bounded above by $(n-1)^{n*n}$
- The number of good puzzle configurations is considerably less (many needles in a *very* large haystack)
- Can't generate and test all, but can search for a good one

# The Search Problem

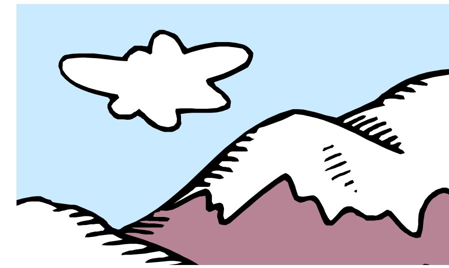
- First, I need a way to rate the maze relative (un)desirability
  - e.g. penalize if goal not reachable from a state
- Then, I need a method for looking around:
  - Start with a *random* maze configuration
  - Change a *random* position to a *random* different jump
  - …but sometimes these changes are counterproductive

# The Drunken Topographer

- Imagine an extremely hilly landscape with many hills and valleys high and low
- Goal: find lowest spot
- Means: airlift a drunk!
- Starts at random spot
- Staggers randomly
- More tired → rejects more uphill steps



# Super-Drunks, Dead-Drunks, and Those In-Between



- The Super-Drunk never tires
  - Never rejects uphill steps
  - How well will the Super-Drunk search?
- The Dead-Drunk is absolutely tired
  - Always rejects uphill steps
  - How well will the Dead-Drunk search?
- Now imagine a drunk that starts in fine condition and very gradually tires.

# What Blacksmiths Knew

- Quenching
  - Heated metal into a cold water barrel
  - Rapid cooling → brittle metal
- Annealing
  - Heated metal allowed to cool slowly
  - Slow cooling → strong metal

# What Statistical Physicists Learned

- Large number of atoms in a random configuration → high energy state
- High temperature (energy input) → atoms reconfigure freely to higher or lower energy states
- Low temperature → atoms reconfigure less freely (usually to lower energy states)
- Metropolis algorithm (Metropolis et al., 1953)

# Rook Jumping Maze Generation

- **State**: a total configuration of jumps
- **Energy**: rating of maze's undesirability
- **Step**: select a random position and change to a different random jump
- The prime design challenge is to define a good energy function, scoring a maze's undesirability.
- What are examples of undesirable characteristics?

# Penalize Unreaching States

- We definitely want to have a solution, and we may want to have all states to have a path to the goal.
- Score: Add 1 per *unreaching* state, i.e. state with no path to goal.

```
[ 1]   1    1    1    2
  4    1    2    3    2
  1    2    2    2    2
  3    3    2    1    1
  2    4    3    2 GOAL

Distances to Goal:
  4    5    4    3    2
  3    4    3    4    2
  4    5    2    4    1
  3    2    2    2    1
  5    6    4    5    0
```

```
score = getNumUnreaching();
```

# Prioritizing Objectives

- First priority: no unreaching states
- Second priority: maximize minimum path length from start to goal
- Find the range of 2nd priority ratings $r$
- Solution: Multiply each 1st priority unit by $(r+1)$
- Example:
  - Max path length less than rows times columns
  - Multiply number of unreaching states by rows times columns, and subtract minimum path length from start to goal

# Reward Longer Solution Paths

- Score: Add rows*cols per *unreaching* state, i.e. state with no path to goal. Subtract minimum path length from start to goal (if path exists).

```
[ 3]   3    4    4    3
  4    1    2    3    2
  3    1    1    1    2
  3    2    3    3    2
  3    4    1    3 GOAL

Distances to Goal:
 18    7   10   17    8
 14    5   12   15   13
  3    4    3    2    1
 19    6   11   18   12
 15    8    9   16    0
```

```
score = rows * cols * getNumUnreaching();
if (getDistance(startState, goalState) != NONE)
    score -= getDistance(startState, goalState);
```

# Possible Considerations

- Should all states be reachable?
- Which structures are/aren't enjoyable challenging?
- Is distance the right measure?  Should number of choice points along the path be used instead?
- Etc. etc. etc. → lots of room for creativity!
- If you can define the measure, we can program the measure.
- Experiment and observe the change in mazes generated

# Summary

- Stochastic local search is a simple, powerful algorithm for finding good configurations in a vast space of configurations, if:
  - One can identify a good “local” step, and
  - One can characterize relative (un)desirability via an energy function.
- It is in the energy function that art and science meet.
  - Energy measure is often non-trivial and requires careful consideration and creativity
- You can become an expert maze designer.
  - Experiment, observe, introspect, express

# Possible Projects

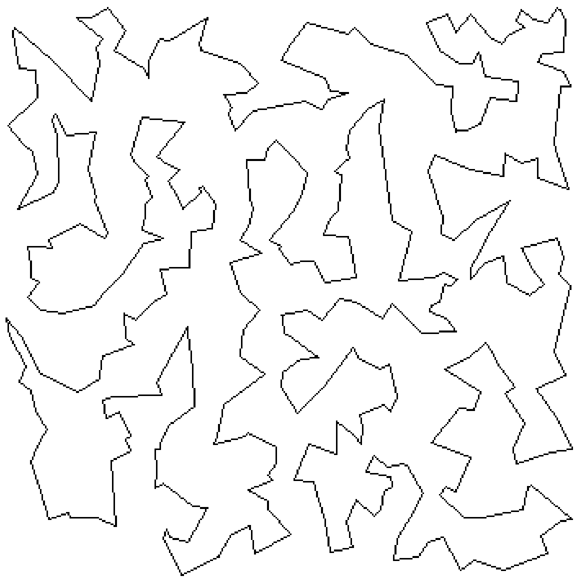
- Free iPhone app
- Publication of paper on maze design
- Public, walkable, student-generated jumping maze
- Daily maze on department website
- Showcase of student work at Celebration 2010
- Puzzle book
- ???  What would be fun for you?  What would add most to your portfolio / resumé?

# References


- Abbott, Robert. *SuperMazes: Mind Twisters for Puzzle Buffs, Game Nuts and Other Smart People*, Prima Publishing, Rocklin, California, 1997.
- Fisher, Adrian. *The Amazing Book of Mazes*, Harry N. Abrams, Inc., New York, 2006.

# Traveling Salesman Problem

- Have to travel a circuit around *n* cities (n = 400)
- Different costs to travel between different cities (assume cost = distance)
- **State**: ordering of cities ($> 8 \times 10^{865}$ orderings for 400 cities)
- **Energy**: cost of all travel
- **Step**: select a portion of the circuit and reverse the ordering



# Sketch of Simulated Annealing

- Pick any starting state
- While gradually cooling the temperature:
  - Randomly change the state
  - Compare old and new energy
  - If new energy lower → accept new state
  - Otherwise accept new state with a probability computed from the *energy change* $\Delta E$ and *temperature* T (probability $e^{-\Delta E/kT}$)