Answer each question carefully in the space provided. There are 6 problems worth 100 points. Point values are given in parentheses to the right of each question. For an additional two points write your initials at the bottom left corner of this page.

Below is a list of languages that are known to be decidable and a list of languages that are undecidable.

Decidable: \(\text{ADFA} \quad \text{ANFA} \quad \text{ABEX} \quad \text{EDFA} \quad \text{EQDFA} \quad \text{ACFG} \quad \text{ECFG}\)

Undecidable: \(\text{ATM} \quad \text{ETM} \quad \text{EQTM} \quad \text{ALLCFG} \quad \text{EQCFG} \quad \text{HALT}_{\text{TM}}\)

1. For each language \(L\) described below, indicate whether \(L\) must be regular, context free, decidable, recognizable, co-recognizable or not recognizable (circle ALL that apply) (3 each)

   a. \(L\) is defined by a CFG.
      regular context free decidable recognizable co-recognizable not recognizable

   b. \(L\) is \(\text{E}_\text{CFG}\).
      regular context free decidable recognizable co-recognizable not recognizable

   c. \(L\) is defined by a Turing machine.
      regular context free decidable recognizable co-recognizable not recognizable

   d. \(L\) is \(\text{E}_\text{DFA}\).
      regular context free decidable recognizable co-recognizable not recognizable

   e. \(L\) is a language for which the pumping lemma for CFGs does not hold.
      regular context free decidable recognizable co-recognizable not recognizable

2. For each of the following statements, indicate the correct proof technique(s) to be used to prove the statement. Circle all that apply. (3 each)

   a. Language \(L\) is decidable.
      direct reduction contradiction construction contrapositive diagonalization

   b. Language \(\text{EQ}_\text{TM}\) is undecidable.
      direct reduction contradiction construction contrapositive diagonalization

   c. Language \(L\) is recognizable.
      direct reduction contradiction construction contrapositive diagonalization

   d. Language \(L\) is not recognizable.
      direct reduction contradiction construction contrapositive diagonalization

   e. Language \(L\) is not context free.
      direct reduction contradiction construction contrapositive diagonalization

   f. Set \(S\) is uncountable.
      direct reduction contradiction construction contrapositive diagonalization

   g. Set \(S\) is countable.
      direct reduction contradiction construction contrapositive diagonalization
3. Suppose we wanted to show the language $L = \{ww^R \mid w \text{ is a palindrome and } \Sigma = \{0,1\} \}$ is not context free using the pumping lemma and $p$ is the pumping length. For each choice of the string $S$ below, indicate if it is a good or bad choice for the proof and why.

a. $S = 10011001$

b. $S = 1^p001^p$

c. $S = 01^p001^p$

4. Fill in each blank with an $A$ or $B$ to make the sentences true. (2 each)

If $A$ is reducible to $B$ and $\text{__________}$ is decidable, then $\text{__________}$ is decidable.

If $A$ is reducible to $B$ and $\text{__________}$ is undecidable, then $\text{__________}$ is undecidable.

5. TRUE or FALSE: Linear Bounded Automata recognize all decidable languages. (1)

6. On another piece of paper, prove TWO of the following. You may use results from the text examples and results you have done for homework. Circle the letter of the problems you are going to prove. (20 each)

a. $\text{EQ}_{PDA} = \{<G,H> \mid G \text{ and } H \text{ are PDAs and } L(G) = L(H)\}$. Prove $\text{EQ}_{PDA}$ is undecidable.

b. $\text{EQ}_{DFA} = \{<A,B> \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$. Prove $\text{EQ}_{DFA}$ is co-turing recognizable.

c. $\text{ALL}_{TM} = \{<M> \mid M \text{ is a TM and } L(M) = \Sigma^*\}$. Prove $\text{ALL}_{TM}$ is undecidable.

d. $\text{UNION}_{DFA} = \{<A,B,C> \mid A, B \text{ and } C \text{ are DFAs and } A = B \cup C\}$. Prove $\text{UNION}_{DFA}$ is decidable.

e. $L = \{<M> \mid M \text{ is a TM and } M \text{ has at least } 5 \text{ states}\}$. Prove that $L$ is decidable.