Induction Proof

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1 Theorem

For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

2 Proof (by mathematical induction)

Let the property $P(n)$ be the equation $1 + 2 + \cdots + n = n(n+1)/2$.

2.1 Basis

Show that the property $P(n)$ is true for $n = 1$.

We must show that $1 = \frac{1(1+1)}{2}$. The right hand side of the equation is $\frac{1(1+1)}{2} = \frac{2}{2} = 1$, which is the same as the left hand side. So the property is true for $n = 1$.

2.2 Induction

Show that for all integers $n = k$, if $P(k)$ is true, then so is $P(k+1)$.

For the induction hypothesis, suppose $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$, for some integer $k \geq 1$. From this we must show that $1 + 2 + \cdots + (k+1) = \frac{(k+1)(k+2)}{2}$.

The left hand side of the equation can be expanded to: $(1 + 2 + \cdots + k) + (k+1)$. Substituting using the induction hypothesis, this is:

$$\frac{k(k+1)}{2} + (k+1)$$

Finding a common denominator and simplifying, we have:

$$\frac{k(k+1)}{2} + (k+1) = \frac{k(k+1)}{2} + \frac{(k+1) \cdot 2}{2} \quad (1)$$

$$= \frac{(k+1)(k+2)}{2} \quad (2)$$

which is what we were trying to show. QED.